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Modeling Premixed Combustion–Acoustic Wave Interactions: A Review

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The interactions between acoustic waves and a premixed combustion process can play an important role in the characteristic unsteadiness of combustion devices. In particular, they are often responsible for the occurrence of self-excited, combustion-driven oscillations that are detrimental to combustor life and performance. A tutorial review is provided of current understanding of these interactions. First, the mutual interaction mechanisms between the combustion process and acoustic, vorticity, and entropy waves are described. Then, the acoustic–flame interaction literature is reviewed, primarily focusing on modeling issues. This literature is essentially organized into four parts, depending on its treatment of 1) linear or 2) nonlinear analyses of 1) flamelets or 2) distributed reaction zones. A sizeable theoretical literature has accumulated to model the unsteady response of the laminar flame structure, for example, the burning rate response to pressure perturbations. However, essentially no serious experimental effort has been performed to critically assess these predictions. As such, it is difficult to determine the state of understanding in this area. On the other hand, good agreement has been achieved between well-coordinated experiments and theory describing the interactions between inherent flame instabilities and acoustically induced flow oscillations. Similarly, both the linear and nonlinear kinematic response of simple laminar flames to acoustic velocity disturbances appear to be well understood, as evidenced by the agreement between surprisingly simple theory and experiment. Other than kinematic nonlinearities, additional potential mechanisms that introduce heat release–acoustic nonlinearities, such as flame holding, or extinction, have been analyzed theoretically, but lack experimental verification. Unsteady reactor models have been used extensively to model combustion processes in the distributed reaction zone regimes. None of these predictions appears to have been subjected to direct experimental scrutiny. It is unlikely that this modeling approach will be useful for quantitative combustion response calculations, due to their largely heuristic nature and the difficulty in rationally modeling the key interactions between reaction rate and the global characteristics of the combustion region, such as its volume. Several areas in need of work are particularly highlighted. These include finite amplitude effects, modeling approaches for interactions outside of the flamelet regime, turbulent flame wrinkling effects, and unsteady vortex–flame interactions.

I. Introduction

THE objective of this paper is to address the manner in which unsteady flowfields and, especially, acoustic disturbances interact with a premixed combustion process. In particular, it focuses on issues associated with modeling these interactions. Such interactions play important roles in the characteristic unsteadiness of turbulent combustion systems found in most processing, power generating, and propulsion applications. The discussion is particularly motivated by the problem of combustion instabilities, which routinely plague the development of combustion systems in industrial processing,[1] solid and liquid rockets,[2,3] ramjets,[4] afterburners, and land-based gas turbines.[5–13] These instabilities arise from interactions between oscillatory flow and heat release processes in the combustor and are manifested as large-amplitude, organized oscillations of the combustor's flowfields. The unsteady heat release generated by the disturbances add energy to the acoustic field when it is in-phase with the pressure oscillations.[14] These oscillations lead to enhanced vibration, reduced part life, flame blowoff or flashback, and even complete system failure. They generally occur at frequencies associated with the combustor's natural longitudinal, radial, azimuthal, or bulk modes. However, they can also be associated with coupled convective–acoustic modes, such as may occur when a convected entropy wave or vortex impinges on the downstream nozzle and excites an acoustic wave.[15] Similar processes arise in pulse combustors, although the enhanced heat or mass transfer generated by the oscillations is desirable in this case.[16,17]

The literature devoted to the associated unsteady combustion problems of combustion noise, inherent flame instabilities, acoustic–flame interactions, and combustion system instabilities is enormous and can only be properly addressed in its entirety by a large volume. As such, this paper makes no attempt to address all of these issues. Rather, its specific objective is to focus on the interactions of premixed flames and acoustic waves without consideration of the larger system in which they occur. Note, then, that many other important issues are not addressed here. These include, first, acoustic characteristics of the overall combustion system. Discussion of linear and nonlinear combustion chamber acoustics may be found in a companion paper in this issue[18] and numerous other references, such as Refs. 1 and 19–21. Second, mechanisms through which these discussed interactions couple with the overall system to become self-exciting are not discussed nor are interactions of acoustic waves with solid fuels,[22] liquid sprays,[23] and non-premixed gaseous and liquid fueled flames.[3,24] In addition, this paper provides only limited discussion of vortex–premixed flame interactions. This is not to downplay their significance; rather, numerous studies have shown that they play critical roles in the oscillatory behavior of premixed systems.[4,9,25–27] However, several thorough reviews on the subject already exist, for example, see Schadow and Gutmark[28] and Coats.[29]

Note several other books and complementary review articles besides those appearing in this issue, which also cover related topics in unsteady combustion and combustion instability. Putnam's[1] work is an excellent overview of combustion-driven oscillations in industrial systems. In addition, see Reynst,[30] Crocco and Cheng,[3] Harrje and Reardon,[24] Yang and Anderson,[31] Natanzon,[32] Zinn,[16] Culick et al.,[33] Markstein,[34] and Williams.[35] In addition, several review articles besides those mentioned also treat related aspects, such as that of Oran and Gardner,[36] which focuses on kinetic–acoustic interaction processes; McManus et al.[37] and Candel,[38,39] which

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discuss mechanisms and active control of instabilities; Culick,[40] which reviews combustion instabilities in liquid-fueled systems; and Clavin,[41] which addresses several topics in acoustically coupled flame instabilities in the more general context of premixed flame dynamics. The preceding list is by no means complete.

The paper is organized in the following manner. The following background sections briefly describe the different regimes of combustion and the associated approaches for modeling acoustic wave interactions with the combustion process in these regimes. It also describes the decomposition of low-amplitude oscillations into canonical acoustic, vortical, and entropy modes. Then, the next subsections summarize the effects of unsteady combustion on the acoustic field and the effect of acoustic oscillations on the combustion process. These sections are meant to be primarily tutorial in nature. Section III, "Literature Review," summarizes the premixed flame–acoustic wave interaction literature. Consistent with the focus of this paper, it primarily addresses issues associated with modeling these interactions, but does include a number of experimental investigations that provide insight into the physical processes that need to be modeled. Section IV, "Flame Transfer Function Calculations," summarizes several analyses of the interactions between laminar flamelets and acoustic waves. The paper then concludes with a discussion of needs for future research and particularly emphasizes the need for more coordinated experimental and theoretical studies of the problem.

II. Background

A. Combustion Regimes

Acoustic wave–flame interactions involve the simultaneous interactions between unsteady kinetic, fluid mechanic, and acoustic processes over a large range of timescales. Fundamentally different physical processes may dominate in different regions of the relevant parameter space, depending on the relative magnitudes of various temporal/spatial scales. The different regimes of interaction between the combustion process and broadband turbulent fluctuations can be readily visualized with the now familiar combustion diagram (Fig. 1).[42] Note that different physical locations of the flame may fall into different regimes on this diagram. The regions denoted by wrinkled and corrugated flamelets correspond to situations where the reactions occur in thin sheets that retain their laminar structure. These sheets become increasingly wrinkled and multiconnected with increasing values of u'/S_L, where u' is the fluctuating velocity and S_L is the laminar flame speed. The region denoted by well-stirred reactor corresponds to the limit where mixing occurs much more rapidly than chemical kinetics and reaction occurs homogeneously over a distributed volume. Note that there is some debate about the characteristics of the combustion process in the regions noted by well stirred reactor and distributed reaction zone.[43]

Consider first the interactions of acoustic waves and flamelets in the wrinkled or corrugated regime. It is useful to consider the ratios of the spatial and temporal scales involved in these interactions. Note first the following length scales: the thickness of a laminar

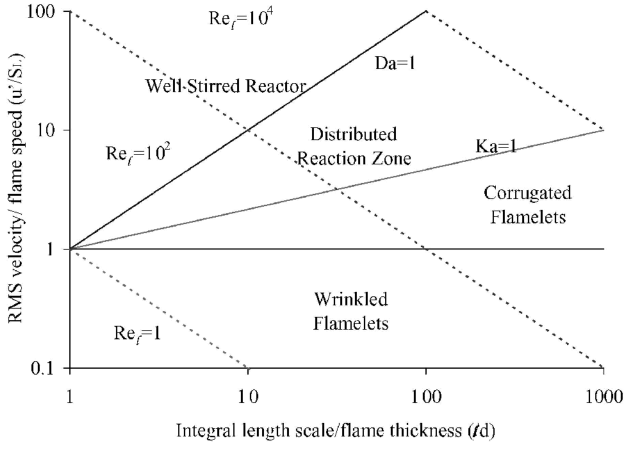

Fig. 1 Classical turbulent combustion diagram.

methane/air flame at standard conditions varies between ~1 and 10 mm (Ref. 44). On the other hand, the acoustic wavelength of a 100, 1000, and 10,000 Hz sound wave at standard conditions is 3.3 m, 33 cm, and 3.3 cm. At higher temperatures, these values are even larger. Given this disparity of length scales, the flame front essentially appears as a discontinuity to the acoustic wave. As such, the fluid dynamics of the flows up- and downstream of the flame can be treated separately from that of the flame structure. The situation is quite different with respect to the relevant timescales. Forming a flame response timescale from the ratio of the laminar flame thickness and flame speed leads to values of ~0.002–0.07 s for methane/air flames. These are of similar magnitude as acoustic perturbations with frequencies between 20 and 500 Hz. Thus, the interior flame structure and, consequently, quantities such as the flame speed do not respond in a quasi-steady manner to acoustic perturbations. This issue will be further addressed in Sec. III.A.1.

The preceding length scale comparison shows that flamelet–acoustic wave interactions can be modeled by treating the flame front as a surface of discontinuity that divides two homogeneous regions. An unsteady analysis of the flame structure provides the necessary jump conditions coupling the flow characteristics in the regions up- and downstream of the flame. Note that this approach was previously developed for studies of the mean flowfield around the flame[45–47] and flame stability,[34] well before its more recent application to the flame–acoustic wave interaction problem.

We will briefly summarize this approach next. More detailed treatments can be found in Refs. 34, 45, 46, and 48. Consider a flame front of arbitrary shape whose instantaneous surface is described by the parametric equation $f(\boldsymbol{x}, t) = 0$. It is assumed that the surface is continuous with a uniquely defined normal at each point. By definition, the following relation always holds on the surface:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \nabla f \cdot \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = 0 \tag{1}$$

With use of this relation, Markstein[34] derives the following kinematic equations relating the flame surface position to the local flow and flame burning velocities:

$$\frac{\partial f}{\partial t} + \boldsymbol{u}_1 \cdot \nabla f - S_1|\nabla f| = 0 \tag{2}$$

$$\frac{\partial f}{\partial t} + \boldsymbol{u}_2 \cdot \nabla f - S_2|\nabla f| = 0 \tag{3}$$

where S and $\boldsymbol{u}$ are the flame speed relative to the gases and flow velocity, respectively. The subscripts 1 and 2 are the value of each quantity on the up- and downstream side of the flame, respectively. Either of the preceding two expressions is often referred to as the G equation in the flame dynamics literature. The flowfields up- and downstream of the flame are coupled across the front by the following relations.[34]

Mass:

$$\rho_1 S_1 = \rho_2 S_2 \tag{4}$$

Normal momentum:

$$p_1 + \rho_1 S_1^2 = p_2 + \rho_2 S_2^2 \tag{5}$$

Tangential momentum:

$$(\boldsymbol{u}_1 - \boldsymbol{u}_2) \times \nabla f = 0 \tag{6}$$

Energy:

$$\rho_1 S_1[h_1 + (\boldsymbol{u}_1 \cdot \boldsymbol{u}_1)/2] = \rho_2 S_2[h_2 + (\boldsymbol{u}_2 \cdot \boldsymbol{u}_2)/2] \tag{7}$$

where ρ and h are density and enthalpy, respectively. The dynamics of the thermodynamic and flow variables up- and downstream of the flame are then described by the mass, momentum, and energy conservation equations. In Sec. IV, we provide an example calculation illustrating the manner in which these equations can be used to calculate the transfer function relating flow perturbations to the flame's heat release response.

It is useful to examine simplified forms of these expressions to gain insight into the relationships between small-amplitude

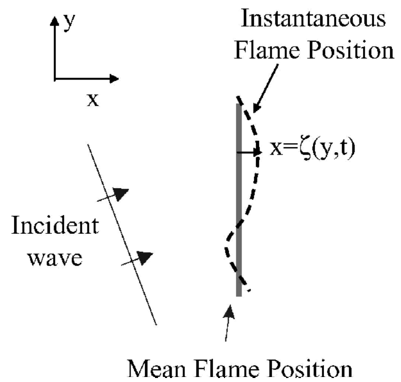

Fig. 2 Schematic of planar flame disturbed by incident acoustic wave.

perturbations across the flame. Assume the flame is a nominally flat, vertically oriented surface in a low Mach number flow whose instantaneous position is described by the equation $x = \zeta(y, t)$ (Fig. 2). It is disturbed by an acoustic plane wave whose wavelength is much larger than the flame thickness. Manipulation of Eqs. (2–7) leads to the following approximate expressions coupling the axial velocity and pressure across the flame[49]:

$$(u_2'/\bar{c}_1) - (u_1'/\bar{c}_1) = (\Lambda - 1)M_s[(S_1'/\bar{S}_1) - (\gamma - 1)(\rho_1'/\bar{\rho}_1)] \quad (8)$$

$$p_2' = p_1' \quad (9)$$

where M_s, γ, and Λ are the laminar flame speed Mach number, specific heats ratio, and mean temperature ratio across the flame. Terms of $\mathcal{O}(M_s^2)$ have been neglected. These expressions show that the unsteady pressure is continuous across the flame, that is, to the order of this approximation, the flame exerts no unsteady force on the gas. However, there is a jump in unsteady velocity across the flame, that is, the flame looks like an acoustic volume source or a monopole. The terms on the right side of Eq. (8) quantifying this jump are related to the flame's unsteady rate of heat release. This jump is directly proportional to the temperature ratio across the flame and the flame speed Mach number, which typically has quite low values (~0.001 for a stoichiometric methane/air flame). Assume a typical acoustic scaling, that is, $p' \sim \bar{\rho} c u' \sim \bar{c}^2 \rho'$; then it can be seen that the second fluctuating term on the right side and the fluctuating velocity terms on the left are of similar magnitude. Thus, this second term results in a velocity increment across the flame that is on the order of M_s and, thus, quite small. The relative magnitudes of the first term on the right side of Eq. (8) and the fluctuating velocity perturbation quantities on the left depends on the specific processes causing the flame speed perturbation. More detailed analyses to be discussed later suggest that flame speed perturbations caused by pressure and/or temperature fluctuations are of similar magnitude $S_1'/\bar{S}_1 \sim \mathcal{O}(p'/\bar{p})$. Thus, acoustic wave amplification induced by the pressure or temperature sensitivity of the flame speed is nonzero, but quite weak. In contrast, flame speed perturbations induced by flow or acceleration perturbations, such as through strain or curvature fluctuations, are potentially much larger because they will scale as $S_1'/\bar{S}_1 \sim \mathcal{O}(u_1'/\bar{u}_1) \sim (1/M)\mathcal{O}(u'/\bar{c}_1)$, that is, they are a factor of the inverse of the mean flow Mach number larger than the already discussed pressure coupling terms. Thus, a velocity or acceleration coupling mechanism is potentially a much stronger source of acoustic amplification.

Flame area fluctuations are another significant source of heat release oscillations, which also arise from a velocity coupling mechanism. The instantaneous differential element of flame surface area is given by

$$\mathrm{d}A_{\mathrm{FL}} = \sqrt{1 + \left(\frac{\partial \varsigma}{\partial y}\right)^2}\,\mathrm{d}y \quad (10)$$

This term is not present in Eq. (8) because of the nominally plane geometry ($\partial \bar{\varsigma}/\partial y = 0$), so that the area perturbation is a second-order quantity in perturbation amplitude. However, as discussed later, it is, in general, a very significant source of acoustic amplification, particularly because the fluctuations in flame area are often generated by velocity fluctuation, that is, $A_{\mathrm{FL}}'/\bar{A}_{\mathrm{FL}} \sim \mathcal{O}(u'/\bar{u})$, thus, leading to a velocity coupled mechanism of acoustic amplification. Analysis in the turbulent combustion literature shows that this same term also plays a leading role in the augmentation of the burning rate in turbulent flows.[50]

More detailed system analysis reveals that the relative role of velocity vs pressure coupled amplification mechanisms is not as straightforward as these arguments might suggest. The reason for this is that flame speed fluctuations induced by velocity/acceleration oscillations, though potentially substantial in amplitude, may be improperly phased with the combustor pressure, for example, the pressure and velocity in an acoustic standing wave is roughly 90 deg, so that the resultant generation of acoustic energy is still quite weak.[51] This result is in contrast to the phasing of the pressure coupled heat release oscillations. These arguments provide insight into general experimental observations that the flame exerts a relatively small amplification of acoustic oscillations in any given cycle of oscillation. Typical measurements suggest amplifications of 1%/cycle.[51] Generally, many cycles of oscillation are necessary for self-excited oscillations to achieve significant overall amplification.

Return to the combustion regime diagram in Fig. 1, and consider next the opposite extreme to the flamelet regime, the well-stirred reactor (WSR) regime. As will be discussed in more detail, several prior studies have suggested that flame–acoustic interactions in this regime can be modeled by generalizing the steady WSR equations to include nonsteady effects. These unsteady reactor equations can be derived from a straightforward spatial integration of the conservation equations over the WSR region by assuming that all spatial quantities are uniform,[35]

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \dot{m}_{\mathrm{in}} - \dot{m} \quad (11)$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \dot{m}_{\mathrm{in}} h_{\mathrm{in}} - \dot{m} h \quad (12)$$

$$\frac{\mathrm{d}M_k}{\mathrm{d}t} = \dot{m}_{\mathrm{in}} Y_{k,\mathrm{in}} - \dot{m} Y_k - \dot{W}_k \quad (13)$$

where $\dot{m}$ and $\dot{W}_k$ are the mass flow rate and consumption rate of the kth species, respectively. M, E, and M_k are the total mass, total energy, and total mass of the kth species in the reactor, respectively. The subscript "in" denotes the inlet value. The steady-state characteristics of the WSR are controlled by the ratio of the chemical kinetic time to the reactor residence time, given by the ratio of the mass flow rate and reactor volume, $\tau_{\mathrm{res}} = \dot{m}/V$. The reactor volume or residence time cannot, in general, be specified by simplified analysis, because it is determined by reaction and mixing rates; prior studies have used experimental and computational analysis to determine these quantities, which are then used as inputs to simplified models.[52]

Note that the preceding equations assume that the perturbations are spatially uniform. If the flame zone is acoustically compact, such an approximation may be adequate to describe acoustic perturbations, but, as will be described next, entropy and vorticity disturbances have much shorter length scales. Fluctuations in these quantities could potentially be of much shorter length scale than the reactor size, indicating that the perturbation variables and flow strain field are spatially distributed in the reactor.

B. Decomposition of Oscillations into Canonical Modes

Consider the characteristics of the oscillations, p', T', ρ', u', etc., in more detail. It is useful to decompose these fluctuations into three canonical types of disturbances[53–55]: vortical, entropy, and acoustic. In other words, each fluctuating quantity can be decomposed as $p' = p_a' + p_v' + p_s'$, $\rho' = \rho_a' + \rho_v' + \rho_s'$, $u' = u_a' + u_v' + u_s'$, and $v' = v_a' + v_v' + v_s'$, where the subscripts a, v, and s denote acoustic, vortical, and entropy disturbances, respectively. Several characteristics of these disturbance modes should be noted.

First, acoustic disturbances propagate with a characteristic velocity equal to the speed of sound, whereas vorticity and entropy disturbances have a characteristic velocity equal to the mean flow velocity with which they are convected. In low Mach number flows, such different characteristic velocities cause these disturbances to have substantially different length scales over which properties vary. Acoustic properties vary over an acoustic length scale, given by $\lambda_a = c/f$, whereas entropy and vorticity modes vary over a convective length scale, given by $\lambda_c = u/f$. Thus, the entropy and vortical mode wavelength is shorter than the acoustic wavelength by a factor equal to the mean flow Mach number. This can have important implications on acoustic–flame interactions. For example, a flame whose length L_{FL} is short relative to an acoustic wavelength, that is, $L_{\mathrm{FL}} \ll \lambda_a$ (acoustically compact) may be of the same order of, or longer than a convective wavelength. Thus, a convected disturbance, such as an equivalence ratio oscillation, may have substantial spatial variation along the flame front and, thus, result in heat release disturbances generated at different points of the flame being out-of-phase with each other. Studies often find that a Strouhal number, defined as $Sr = f L_{\mathrm{FL}}/u$, is a key parameter that effects the flame response to perturbations; note that Strouhal number Sr is simply a ratio of the flame length and convective wavelength, $Sr = L_{\mathrm{FL}}/\lambda_c$. A flame whose length is much less than an acoustic convective wavelength is referred to as acoustically convectively compact.

Second, acoustic disturbances, being true waves, reflect off boundaries, are refracted at property changes, and diffract around obstacles. Entropy and vorticity disturbances, on the other hand, are simply convected by the mean flow and diffuse from regions of high to low concentration. Thus, an acoustic wave impinging on the open end of a pipe may reflect back toward its point of origination with almost equal magnitude as that of the incident wave, whereas the entropy and vorticity disturbances simply convect away. (Note, however, that entropy disturbances incident upon a nozzle do generate acoustic waves.)

Third, acoustic disturbances are associated with irrotational, volumetric fluctuations. Thus, the divergence of the acoustic velocity field is nonzero, whereas its curl is zero, that is, $\nabla \cdot \boldsymbol{u}_a' \neq 0$ and $\nabla \times \boldsymbol{u}_a' = 0$. In contrast, vortical fluctuations are rotational but incompressible, that is, $\nabla \cdot \boldsymbol{u}_v' = 0$ and $\nabla \times \boldsymbol{u}_v' \neq 0$. Velocity oscillations due to entropy disturbances are generally negligible; rather, entropy fluctuations are primarily manifested by density and temperature disturbances.

Fourth, in a homogeneous, uniform flow, these three disturbance modes propagate independently in the linear approximation. However, finite amplitude disturbances do interact, for example, the interaction of two finite amplitude vortical disturbances generates an acoustic disturbance.[53] Coupling between small-amplitude perturbations occurs at boundaries or in regions of inhomogeneity. For example, consider a small-amplitude acoustic wave impinging obliquely on a rigid wall. Enforcement of the no-slip boundary condition requires that the tangential vortical and acoustic velocity components sum to zero at the wall. Thus, the incident acoustic wave excites vorticity disturbances near the wall whose magnitude is determined by the requirement that the sum of their tangential components is zero. This vorticity wave has a significant magnitude over a normal distance associated with the acoustic boundary-layer thickness.

C. Effect of Oscillations on the Flame

Disturbances in the flow and acoustic field exert influences on the flame in a variety of ways, each of which constitutes a potential mechanism for self-excited oscillations. This section briefly summarizes the basic manner in which oscillations disturb the combustion process to emphasize the number of different ways through which such disturbances may occur. As noted by Clanet et al.,[56] they can be classified into different categories depending on whether they modify the local internal structure of the flame (such as the local burning rate) or its global geometry (such as its area). With reference to the earlier discussion in Sec. II.A, the former and latter categories are often associated with pressure and velocity coupled mechanisms, respectively.

The most basic effect of an acoustic wave on a flame is simply causing a fluctuation in the mass flow rate of reactive mixture into the flame. This mass flow fluctuation is due to both the velocity oscillations, which carry the mixture toward the flame, or density oscillations, which affect the mass of reactive mixture per unit volume.

Also, because the flame's position and orientation depends on the local burning rate and flow characteristics, velocity perturbations cause wrinkling and movement of the flame front. In turn, this modifies its local position and curvature, as well as its overall area or volume. These velocity disturbances can be acoustic or vortical in nature and, thus, propagate at the sound or flow speeds, respectively. To illustrate the disturbance of a flame by an acoustic velocity disturbance, Fig. 3 shows a photograph from Ducruix et al.[57] of a simple Bunsen flame disturbed by acoustic flow oscillations generated by a loudspeaker placed upstream of the flame. Figure 3 clearly shows the large distortion of the flame front, which is evidenced by the pronounced cusp in the center of the flame. This flame disturbance is convected downstream by the mean flow, so that it varies spatially over a convective wavelength. Flame perturbations that are generated by vortical disturbances are generally associated with large-scale vortical structures that are due to hydrodynamic flow instabilities. Figure 4 shows a simulated result of such an interaction,[58] where the flame is disturbed by vortex structures that are periodically shed off the rapid expansion. Whether the flame is disturbed by acoustic or vortical velocity disturbances, the distortion of the flame front results in heat release oscillations. These heat release oscillations are generally attributed to the variation in flame surface area,[59] although they could also originate from fluctuations in the burning rate due to strain rate fluctuations or even local extinction. In

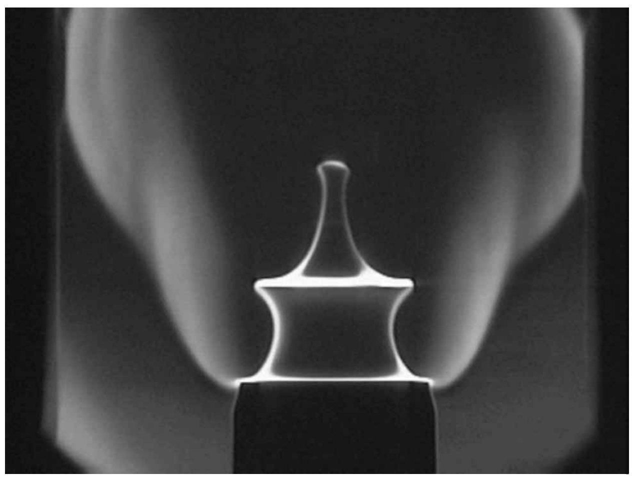

Fig. 3 Photograph of flame disturbances generated by acoustic velocity oscillations (courtesy of S. Ducruix, D. Durox and S. Candel, Centre National de la Recherche Scientifique and Ecole Centrale de Paris).

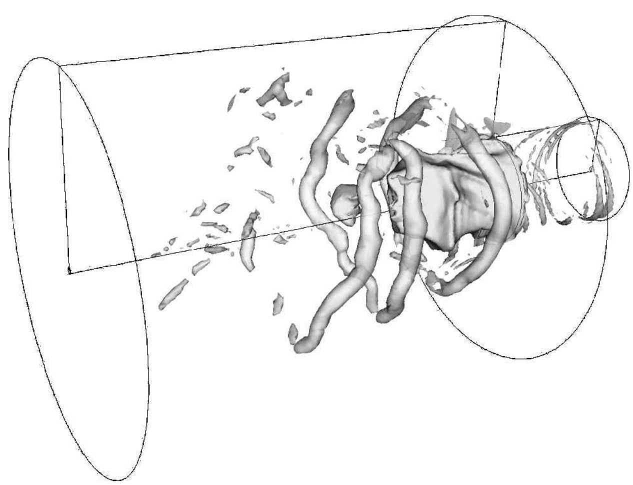

Fig. 4 Computation of flame disturbed by vortical structure (courtesy of S. Menon and C. Stone).

addition to this flame area modulation mechanism, coherent vortical structures also cause heat release oscillations through large-scale entrainment of the reactive mixture, which reacts in a sudden burst, such as when the vortex impinges on a wall.[28,60]

The flame's burning rate is also sensitive to the perturbations in pressure, temperature, and strain rate that directly accompany the acoustic wave. As such, the local rate of heat release per unit area of flame or volume of reactor oscillates in time, even if the total surface area or volume of the combustion process remains fixed. Furthermore, dynamic effects can cause this response to exceed its quasi-steady value by an order of magnitude.

Besides the direct effect of an acoustic oscillation on the burning rate and/or flame position, acoustic pressure and velocity oscillations can also exert indirect effects. For example, disturbances in the premixing section of the combustor generate oscillations in the fuel/air ratio of the reactive mixture.[61] This fuel/air ratio disturbance can be generated either by a fluctuating fuel flow rate (such as by fluctuating the pressure drop across the fuel nozzle) or the fluctuating airflow rate due to acoustic velocity perturbations. These fuel/air ratio disturbances are convected by the mean flow (and, thus, have an associated convective wavelength) and disturb the flame.[62]

D. Effect of Heat Release on Oscillations

Having briefly considered the different ways in which flow oscillations cause combustion process disturbances, we now consider the related problem of the manner in which combustion process disturbances effect or generate flow oscillations. This mutual interaction is ultimately what is responsible for self-excited, combustion-driven oscillations.

Oscillations in heat release generate acoustic perturbations. This sound generation is manifested as the broadband combustion roar of turbulent flames[63,64] and, in the context of combustion instabilities, by discrete tones. In terms of sound generation, a flame can be thought of as a distribution of monopoles whose local source strength is proportional to the unsteady rate of heat release. The fundamental mechanism for this sound generation is the unsteady gas expansion as the mixture reacts.

To illustrate, consider a combustion process in a free-field environment. Equation (14) is an expression for the far-field pressure radiated by the unsteady heat release.[63] The effects of temperature gradients and mean flow, which cause additional reflection, convection, and refraction of sound, are not included in this expression.

$$p'(\boldsymbol{x}, t) = \frac{\gamma - 1}{4\pi \bar{c}^2} \int_{x_s} \frac{1}{|\boldsymbol{x}_s - \boldsymbol{x}|} \frac{\partial q'(\boldsymbol{x}_s, t - |\boldsymbol{x}_s - \boldsymbol{x}|/\bar{c})}{\partial t} \, \mathrm{d}\boldsymbol{x}_s \tag{14}$$

where $q'(\boldsymbol{x}_s, t)$, $\boldsymbol{x}$, and $\boldsymbol{x}_s$ are the unsteady rate of heat release per unit volume, observer location, and combustion region, respectively. This expression shows that the acoustic pressure at the observation point $\boldsymbol{x}$ is related to the integral of the unsteady heat release over the combustion region, at a retarded time $t - |\boldsymbol{x}_s - \boldsymbol{x}|/\bar{c}$ earlier. If the combustion region is much smaller than an acoustic wavelength, that is, acoustically compact, then disturbances generated at different points in the flame arrive at the observer point with essentially the same retarded time. In this case, Eq. (14) takes the form

$$p'(\boldsymbol{x}, t) = \frac{\gamma - 1}{4\pi \bar{c}^2 R_o} \frac{\partial}{\partial t} Q'\left(t - \frac{R_o}{\bar{c}}\right) \tag{15}$$

where

$$Q'(t) = \int_{x_s} q'(\boldsymbol{x}_s, t) \, \mathrm{d}\boldsymbol{x}_s \tag{16}$$

and R_o is the average distance between the combustion region and observer. Equation (15) shows that in the compact flame case, the distribution of the heat release is unimportant; what matters is the total, spatially integrated value.

The effect of unsteady heat release in a confined environment is related, but has some key differences. As earlier, a simple expression can be derived in the case where the flame region is acoustically compact. It is most convenient to express this effect in terms of jump conditions that relate the acoustic perturbations across the flame region:

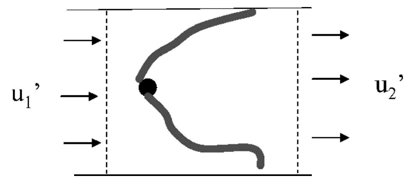

Fig. 5 Schematic of ducted flame, illustrating velocity jump induced by unsteady heat release.

$$u_2' - u_1' = (1/A_d)[(\gamma - 1)/\gamma \bar{p}]Q' \tag{17}$$

where Q' has the same definition as earlier and A_d is the cross-sectional area of the duct. Note that this expression assumes that the velocity is evaluated at locations far enough up- and downstream of the flame that it is one dimensional. It shows that unsteady heat release causes a jump in acoustic velocity across the flame (Fig. 5). This result could be expected, given the unsteady gas expansion that results from the heat release disturbance. Note that this expression is valid only when the flame region is compact; if it is not, waves originating from different regions of the combustion process may destructively interfere. Incorporating noncompactness effects in either the free-field or ducted case can be accomplished using a multipole expansion procedure.[65,66]

Besides generating acoustic waves, unsteady heat release also generates entropy disturbances. Approximate jump conditions similar to those already given can also be derived. However, deriving these expressions requires the much more restrictive assumption of a convectively compact flame zone and is not pursued here.

Whether these entropy disturbances significantly affect the dynamics of the combustor depends on the downstream configuration. If the combustor area remains relatively constant so that the flow passes out of the system unrestricted, such as an open ended pipe, the disturbance will simply convect out of the system and be dissipated in the atmosphere. This behavior is in contrast to the acoustic disturbance, which is usually strongly reflected by such a boundary. However, if the flow is accelerated, for example, by passing through a nozzle, the entropy disturbance generates sound.[67] This sound-generation mechanism is dipole in nature and due to the acceleration of the density disturbance, generating an unsteady force on the gas.

Even in the absence of heat release perturbations, the presence of steady heat release introduces important coupling between the acoustic, vortical, and entropy modes. First, an acoustic oscillation incident on a flame generates entropy and vorticity disturbances.[34] The vorticity disturbance is generated through at least two mechanisms. Probably most significant is the baroclinic mechanism, which occurs if the wave is obliquely incident on the flame. It is due to the misalignment of the mean density gradient and the fluctuating pressure gradient, that is, $\nabla\bar{\rho} \times \nabla p' \neq 0$. Also, the unsteady wrinkling of the flame front by the acoustic perturbations causes additional unsteady vorticity generation.[45]

Entropy and vorticity disturbances impinging on a flame excite acoustic waves if their phase speed along the flame front (not the flow speed) is supersonic.[34] In low Mach number flows, this can occur if the flame is nearly orthogonal to the flow.

E. Inherent Premixed Flame Instabilities

Even in the absence of acoustic oscillations, premixed flames may be unsteady because of intrinsic instabilities. These instabilities are significant because their interaction with externally imposed acoustic oscillations can result in qualitative changes in the flame's dynamics. We briefly introduce these instabilities; detailed discussions and analysis may be found by Williams[35] or Clavin[68].

Williams[35] suggests that intrinsic premixed flame instabilities with one-step chemistry can be grouped into three basic categories: body force, hydrodynamic, and diffusive–thermal. The body force instability is analogous to the classical buoyant mechanism where a heavy fluid resting above a lighter one is destabilized by the action of gravity. In the same way, flames propagating upward divide a higher and lower density region and, thus, are unstable. Similar instabilities can be induced by acceleration of the flame sheet, either through a variation of the burning velocity or an externally imposed flow perturbation. As will be discussed further in the following section, the latter mechanism plays an important role in certain acoustic–flame interaction phenomenon, where the acceleration is provided by the acoustic velocity field.

The hydrodynamic, or Darrius–Landau instability, has an underlying mechanism that is purely hydrodynamic in nature. Specifically, a front dividing two gases of different density that propagates at a constant velocity normal to itself with respect to the more dense gas is unstable for all wavelengths of perturbation.[35] This mechanism is due to gas expansion across the flame, which causes the incident flow streamlines to diverge/converge in front of a flame disturbance that is convex/concave to the unburned gas. The resulting flow divergence/convergence causes the flow to locally decelerate or accelerate, respectively, causing the disturbance to further grow. Whereas initial analyses that assumed constant flame speed and neglected gravity effects found this mechanism to be intrinsically destabilizing, more recent analysis has modified this result. Specifically, the dependence of the local burning velocity on the radius of flame curvature can stabilize short wavelength perturbations. Longer wavelength perturbations are stabilized by gravity for downward propagating flames.

The diffusive–thermal instability is due to the effect of flame front curvature on the rate of diffusion of heat and reactive species. For example, a disturbance that causes the front to bulge toward the unburned gas results in defocusing of conductive heat flux that heats the incoming mixture. In the same way, it results in focusing of the diffusive flux of the deficient reactant into the flame. If the heat conductivity and limiting reactant diffusivity are equal, that is, a unity Lewis number, $Le = D_T / D_M$, then these effects balance so that the burning velocity is unaltered. For mixtures with Lewis numbers less than about unity, this mechanism is destabilizing. In addition, in multiple reactant systems, variations in the relative diffusion rates of reactants can introduce variations in mixture composition at the flame, also causing instability.

III. Literature Review

Now that some background on the mutual interactions between acoustic waves and premixed flames has been provided, the following section reviews prior work on the subject. The literature on the problem treats a variety of experimental hardware and modeling approaches that address the full spectrum of very fundamental, idealized configurations to realistic geometries. The organization of these studies is as follows. First, studies of the response of flames to low-amplitude disturbances, such as linear theoretical analyses, are described in Subsecs. III. A and III.B. Flame response analyses and measurements of large amplitude perturbation effects follow in Subsec. III.C. In addition, the low-amplitude response studies are further grouped into the two sections, "Fundamental Studies" and "Realistic Geometries." Finally, the "Fundamental Studies" section is further subdivided by the combustion regime of consideration, flamelets, or WSRs.

A. Fundamental Studies: Linear Response

1. Flamelet Studies

A large number of studies have considered various fundamental aspects of the acoustic wave–premixed flamelet interaction problem.

Manson[69] appears to have first calculated the reflection and transmission coefficients of a planar flame. The analysis is quite similar to Rayleigh's[14] study of wave reflections from a temperature discontinuity. Chu[70] performed a more detailed investigation of a similar problem; his investigated geometry is shown in Fig. 2. It consists of an infinitely long, planar front that is disturbed by a normally impinging acoustic wave. Chu used a linearized version of the matching conditions given in Eqs. (4–7) and did not consider the internal flame structure. He calculated the reflection and transmission coefficients of a planar flame front and also showed that acoustic waves are generated or amplified by changes in flame speed, heat of reaction of the reactive mixture, entropy of the incoming mixture, or specific heats ratio.

This result was generalized by Lieuwen,[49] who considered the interaction between an obliquely incident acoustic wave on a flame front. This generalization introduces the additional phenomena of flame wrinkling and vorticity production into Chu's problem.[70] In addition, he[49] calculated the net acoustic flux out of the flame, thereby allowing for a calculation of the flame's amplification or damping of the disturbance. Energy added to the acoustic field by unsteady heat release processes results from the unsteady flux of unburned reactants through the flame by fluctuations in either the flame burning velocity or density of the unburned reactants. Acoustic energy is dissipated by the transfer of acoustic energy into vorticity fluctuations that are generated at the flame front by the baroclinic vorticity production mechanism. Depending on the temperature ratio across the flame, magnitude and phase of the flame burning velocity response, and angle of incidence between the wave and the flame, the acoustic disturbance could be damped or amplified.

A number of analyses, such as those of McIntosh,[71] McIntosh et al.,[72] Peters and Ludford,[73] Van Harten et al.,[74] Keller and Peters,[75] and Ledder and Kapila,[76] have analyzed the internal structure of a flat flame perturbed by an acoustic wave using high activation energy asymptotics and single-step kinetics. These results quantify the response of the flame's burning velocity to the unsteady pressure and temperature variations in the incident wave. Many of these results are summarized by McIntosh,[77] who emphasizes the different characteristics of the interaction depending on the relative magnitudes of the length and time scales of the acoustic wave and flame preheat and reaction zone. Following McIntosh,[77] define the following ratios of these length scales and timescales:

$$\tau \equiv \frac{\text{diffusion time}}{\text{acoustic period}}, \qquad N \equiv \frac{\text{acoustic wavelength}}{\text{diffusion length}} \tag{18}$$

These ratios are related by the Mach number of the flame burning velocity,

$$M_s = S_L / c_u = 1/\tau N \tag{19}$$

Also, define the dimensionless overall activation energy,

$$\theta = E_a / R T_b \tag{20}$$

where E_a is the overall activation energy, R is the gas constant, and T_b is the burned gas temperature. He identifies four different regimes based on the relative magnitudes of these parameters.

1) $N \gg 1/M_s$, that is, $\tau \ll 1$; acoustic wavelength is much longer than the flame thickness and the flame responds in a quasi-steady manner to acoustic disturbances.

2) $N \sim \mathrm{O}(1/M_s)$, that is, $\tau \sim \mathrm{O}(1)$; acoustic wavelength is much larger than the flame thickness, but acoustic and flame response times are commensurate.

3) $N \sim \mathrm{O}(1/\theta^2 M_s)$, that is, $\tau \sim \mathrm{O}(\theta^2)$; fast timescale acoustic oscillations affect inner reaction zone. Spatial pressure gradients are not important in combustion zone.

4) $N \sim \mathrm{O}(1)$, that is, $\tau \sim \mathrm{O}(1/M_s)$; pressure gradients occur over same length scale as flame thickness.

The regime of most interest to unstable combustors is likely cases 1 and 2. For example, a frequency of 400 Hz roughly corresponds to a $\tau \sim 1$ value in a stoichiometric methane/air flame. For these cases, McIntosh derives the following expression relating the mass burning rate and acoustic pressure perturbation[71]:

$$\left(\frac{m'}{\bar{m}}\right) \bigg/ \left(\frac{p'}{\bar{p}}\right) \equiv \upsilon = \frac{2\theta(\gamma - 1)}{\gamma} \times \frac{(-i\tau)(s - 1 + 1/\Lambda)}{[\theta(\Lambda - 1)/\Lambda][Le(s - 1) + (1 - r)] - 2s(1 - r)} \tag{21}$$

where we assume an $\exp(-i\omega t)$ time dependence, Le is the Lewis number, and

$$s = \sqrt{1 - 4i\tau/Le}, \qquad r = \sqrt{1 - 4i\tau} \tag{22}$$

Figure 6 shows the dependence of the normalized mass burning rate response ν. It increases roughly linearly with θ and as the square root of dimensionless frequency τ and flame temperature jump, Λ. The Lewis number dependence is quite weak for Lewis number values near unity. This result illustrates that the mass burning rate response is substantially larger than its quasi-steady value in the physically interesting $\tau \sim O(1)$ case. Although these analyses are most relevant to the flamelet combustion regimes, McIntosh suggests that they could also be applied to the distributed reaction regime, where the laminar flame thickness is replaced by that of the thickened preheat zone.

Although not specifically addressing the acoustic problem, related work to determine the response of the mass burning rate to unsteady stretch has been studied analytically by Joulin[78] and Huang et al.[79] and computationally by Im and Chen.[80] In the steady case, linearized analysis suggests that the burning velocity dependence on curvature and hydrodynamic stretch combines into a single term.[68] However, Joulin's[78] analysis predicts that these two terms have a different frequency response in the unsteady case; the unsteady strain effect diminishes with frequency, whereas the unsteady curvature term is independent of frequency. The former prediction is consistent with Im and Chen's[80] calculations, which predicted that the flame speed response to strain rate fluctuations attenuates as the frequency exceeds the inverse of the characteristic flame response time (Figure 7).

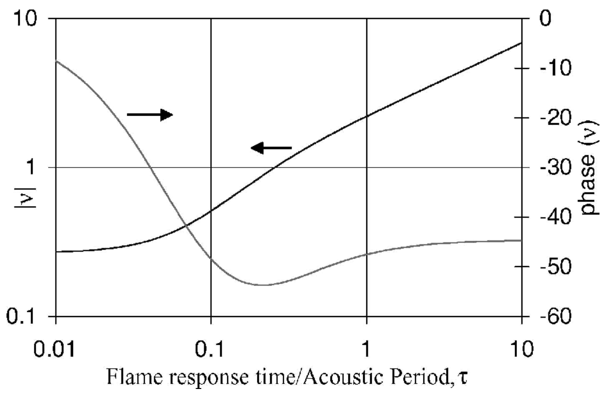

Fig. 6 Normalized mass burning rate response to acoustic pressure perturbations ν (adapted from McIntosh[71]).

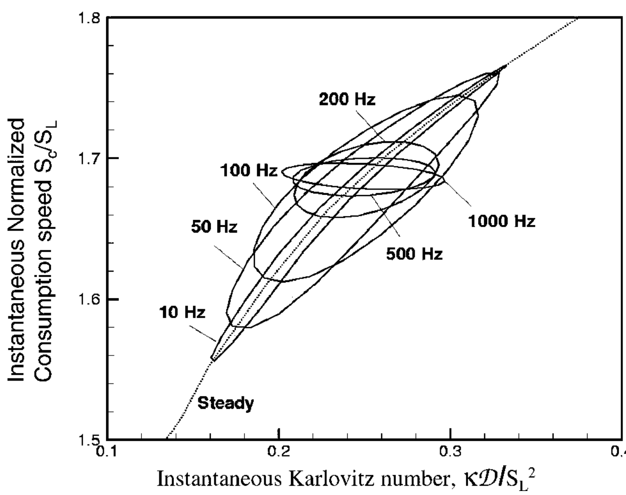

Fig. 7 Dependence of instantaneous flame consumption speed S_c on instantaneous Karlovitz number at several frequencies of oscillation; calculation performed at $\phi = 0.4$ for a hydrogen/air flame, where $\mathcal{D}$, k, and S_L are the diffusivity of oxygen, instantaneous stretch rate, and unstretched laminar flame speed, respectively (courtesy of Im and Chen[80]).

Experiments showing that flames propagating downward from an open end of a tube emit spontaneous acoustic oscillations[81–83] has also motivated a number of analyses, such as those of Clavin et al.,[84] Markstein,[34,85] Clanet et al.,[56] Searby and Rochwerger,[86] and Pelce and Rochwerger.[87] Similar behavior has also been reported in a Taylor–Couette combustor by Vaezi and Aldredge.[88] In a typical experiment,[83] the reactive mixture is ignited at the open end of a vertical tube. Photographs obtained by Aldredge of the resulting sequence of flame characteristics are shown in Fig. 8. As the flame propagates down the tube, it develops a cellular shape due to the inherent Darrieus–Landau instability (Figure 8a). For sufficiently lean mixtures with low flame speeds, the flame propagates down the tube without generating sound. Searby notes, however, that for flames with burning velocities in the 16–25 cm/s range, a primary acoustic instability occurs.[83] Measurements and analysis indicate that this primary instability is generated by acoustic oscillations with a frequency associated with a natural acoustic mode of the duct that modulates the area of the cellular structures and, thereby, the heat release rate. These oscillations grow rapidly and eventually result in a remarkable restabilization of the flame front, where the cellular structure disappears and the flame reverts to a nearly planar front (Fig. 8b). Measurements indicate that the flame's propagation speed slows down substantially due to the reduction in surface area and has a value that is close to the laminar burning velocity.[83] In addition, the growth rate of the oscillations reduces markedly. Analysis indicates that this behavior is due to stabilization of the

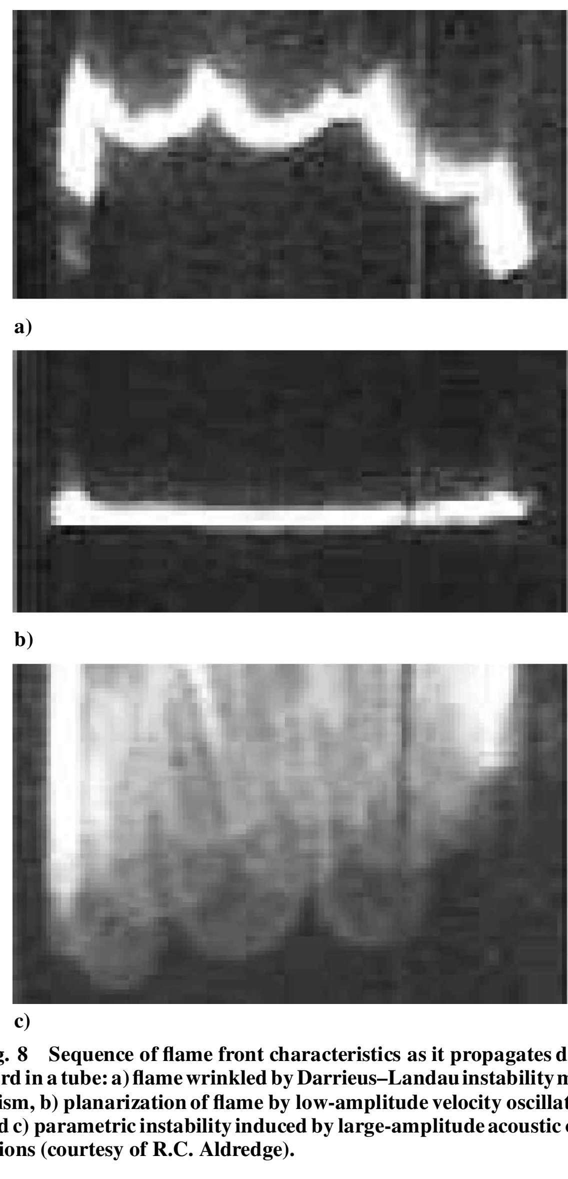

Fig. 8 Sequence of flame front characteristics as it propagates downward in a tube: a) flame wrinkled by Darrieus–Landau instability mechanism, b) planarization of flame by low-amplitude velocity oscillations, and c) parametric instability induced by large-amplitude acoustic oscillations (courtesy of R.C. Aldredge).

Darrieus–Landau flame instability by the oscillatory acceleration imposed by the acoustic field (see Refs. 86 and 89).

For flames with laminar burning velocities greater than about 25 cm/s, this primary instability is followed by a more violent secondary instability with an even larger growth rate than observed in the very initial stages (Fig. 8c). The nearly planar flame develops small, pulsating cellular structures whose amplitude increases rapidly. These cellular structures oscillate at half the period of the acoustic oscillations. This parametric acoustic instability is due to the periodic acceleration of the flame front by the unsteady velocity field, which separates two regions of differing densities. With increased amplitudes, these organized cellular structures break down into a highly disordered, turbulent front.

In the case where the ambient flowfield is highly turbulent, Vaezi and Aldredge[90] did not find any evidence of occurrence of a primary acoustic instability. However, they did note that the parametric instability still appeared. In addition, they noted that for sufficiently high-turbulence levels, the appearance of the parametric instability did not result in additional acceleration of the flame front. This is in contrast to the case where the ambient flowfield is quiescent, where the parametric instability results in substantial flame front acceleration.

Markstein[34] first recognized that the period doubling behavior occurring during the parametric instability was indicative of a parametrically pumped oscillator. He showed that this behavior could be described by the following parametric oscillator equation which he derived on phenomenological grounds:

$$A\frac{\mathrm{d}^2 y(k,t)}{\mathrm{d}t^2} + B\frac{\mathrm{d}y(k,t)}{\mathrm{d}t} + [C_0 - C_1\cos(\omega t)]y(k,t) = 0 \quad (23)$$

where A, B, and C are coefficients defined in Ref. 86, k is the wave number of the perturbation, and ω is the frequency of imposed oscillation. This equation has subsequently been derived from a rigorous application of laminar flame theory.[86] The damping coefficient B is always positive, whereas the coefficient C_0 is negative if the planar flame front is nominally unstable. In the case where C_0 is negative, this equation has the properties that the solution is unstable in the absence of imposed oscillations, that is, $C_1 = 0$, is stabilized in the presence of small but finite amplitude perturbations and is destabilized in the presence of large-amplitude parametric oscillations. Searby and Rochwerger[86] and Bychkov[89] have shown that the predictions of these analyses are in excellent agreement with measurements.

The cited analyses primarily focused their attention on nominally flat, laminar flames. Other studies have investigated the characteristics of wave interactions with wrinkled, turbulent flames[91,92] with randomly moving fronts. The existing theory is restricted to high-frequency disturbances because it requires that the radii of flame wrinkling be large relative to the acoustic wavelength. It predicts that a coherent, harmonically oscillating wave incident on a flame generates scattered coherent and incoherent disturbances. The incoherent disturbances have a distributed spectrum that is roughly symmetric about the incident wave frequency f_i. These incoherent disturbances are due to the randomly moving flame front, resulting in Doppler shifted scattered waves. These predicted qualitative characteristics have been confirmed in subsequent experiments (Figure 9), which reveal that the scattered wave spectra has a narrow band peak at the incident wave frequency with distributed side bands.[93]

It is the characteristics of the coherent reflected and transmitted waves that are of most interest to this paper. The theory predicts that the wrinkled characteristics of the flame act as a source of damping of coherent acoustic energy. This energy is fed from the coherent to the incoherent field. The mechanism for this damping is primarily kinematic in nature as the phase of the scattered waves differ from point to point along the flame front because of differences in distance the wave travels before impinging on the flame and reflecting. Phase mismatch between disturbances originating from different points of the flame results in destructive interference between these different waves. In general, the theory predicts that the characteristics of the scattered field depend on the statistical distribution of the flame front about its average position. In the limit where the scales of flame wrinkling are much smaller than a wavelength, only the variance of the flame position, that is, the turbulent flame brush thickness, is important and the coherent scattered field is damped by the amount $1 - 2(k\sigma\cos\theta_i)^2$, where k, σ, and θ_i are the acoustic wave number, flame brush thickness, and relative angle between the incident wave and average flame position. Besides the reduction in amplitude, the coherent field has a phase offset relative to its smooth surface value if the flame position is asymmetrically distributed about its mean position. This result is of lower order than the amplitude effect, however, because it is proportional to $(k\sigma\cos\theta_i)^3$. Although this predicted damping of coherent oscillations has not been experimentally verified, experiments have confirmed the complementary theoretical prediction that the scattered power in the incoherent field increases with decreasing acoustic wavelength for $k\sigma$ values less than or near unity and saturates for large $k\sigma$ values, (Fig. 10).

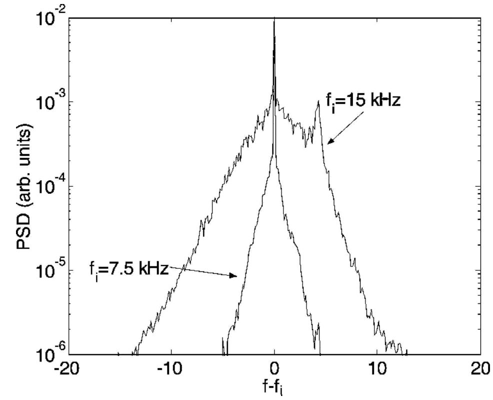

Fig. 9 Measured spectra of scattered field excited by 7.5 and 15 kHz incident waves (adapted from Lieuwen[93]).

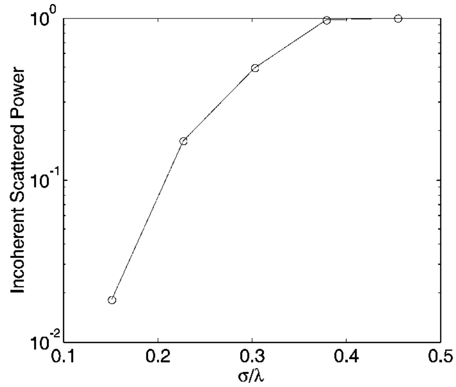

Fig. 10 Dependence of scattered incoherent power on ratio of flame brush thickness σ and acoustic wavelength λ (adapted from Lieuwen[93]).

We conclude this section with several summary remarks. The interactions of an acoustic wave with a planar flame has received extensive theoretical attention (e.g., Refs. 49 and 69–80), particularly in regard to the flame's chemical kinetic, pressure coupled response. However, no complementary experimental investigations to assess these predictions critically appear to have been undertaken. This is due to the difficulty of actually assembling an experiment that emulates the simple, fundamental geometries that are most amenable to analytical attack, for example, analyses typically assume infinitely long flat flames or that reflected and transmitted waves propagate away without subsequent reflections. Thus, in any real experiment, wave diffraction from flame edges or wave reflection from hardware must be accounted for. Given the very slight amplification, on the order of 1%, that an acoustic wave may experience due to flame interactions, small errors in correcting for reflection, diffraction or other effects can easily render any quantification of wave amplification completely useless. Nonetheless, these inherent difficulties do not detract from the reservation that must be placed on a fairly mature body of theory that has not been subjected to experimental scrutiny. Researchers and theoreticians need to devise experiments to circumvent the difficulties highlighted, as well as theory that is more amenable to realization in the laboratory. Some resolution to this problem could also be achieved by detailed computational analyses, which could circumvent many of the difficulties noted earlier.

On the other hand, the problem of acoustic wave interaction with inherent instabilities of a nominally flat flame appears to be well understood. This is one of the few problems in the field where good quantitative agreement has been achieved between experiments and theory.

2. *Distributed Reaction Models*

We next consider combustion–acoustic interactions in the WSR regime (Fig. 1). Unsteady reactor models are routinely used to study kinetically driven instabilities in multistep chemical mechanisms and extinction and ignition phenomenon.[94,95] The interactions between acoustic oscillations and a distributed reaction zone has been similarly modeled. Richards et al.[96] and Janus and Richards[97] used such models to describe the dynamics of a pulse combustor and a lean, premixed combustor. Using the unsteady WSR equations, they determined the response of the unsteady heat release to perturbations in pressure, reactants mass flow rate, or equivalence ratio. The model predicted the dependence of combustor stability behavior on mean operating parameters, such as inlet temperature or flow rate. Using a similar model, Lieuwen et al.[98] calculated the unsteady heat release response to equivalence ratio perturbations. These analyses assumed that the reactor residence time was fixed, that is, oscillations in reactive mixture composition or temperature did not affect the residence time. The effects of neglecting these variations could be substantial in certain cases, however, as will be discussed here and in Sec. IV.

Park et al.[99] studied the dynamics of a reactor with a fluctuating residence time. They show that the phase between reactor residence time and heat release oscillations qualitatively changes above and below the point of maximum reactor heat release. They present an intuitive method of explaining this result (Fig. 11). The steady-state reactor conditions are determined by the point where the rate of heat release by chemical reaction equals the net rate of convection of energy out of the reactor. The dependence of these two rates upon reactor temperature are indicated by the solid and dashed lines in Fig. 11, respectively. Energy convection rate curves are drawn at several residence times. Note that maximum reaction rate occurs at a certain value of residence time. Consider the effect of small perturbations in residence time at mean residence times above and below this maximum value. As indicated in Fig. 11, these perturbations result in reaction rate oscillations that are out-of- and in-phase with the residence time perturbation. Thus, Park et al.[99] note that, as the combustion process approaches the blowout point, it will pass through this point of phase reversal. It can be anticipated that a qualitative change in combustion dynamics will result. Whereas the authors original analysis utilized one-step kinetics, they have also

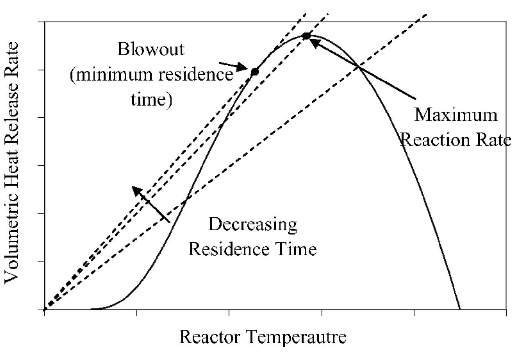

Fig. 11 Steady-state reactor solution occurs at high-temperature intersection of two points; Dependence of ——, rate of reaction and - - - -, convection on reactor temperature (after Park et al.[99]).

performed a similar analysis on a four-step reduced mechanism and obtained similar results.[100]

To extend the reactor approach to situations where the flame was convectively non-compact and, thus, flow disturbances varied substantially over the flame region, Lieuwen et al.[62] treated the combustion zone as a distribution of infinitesimal, independent reactors whose input conditions were given by that of the local flow at the associated location. Although this heuristic treatment allowed for a consideration of important noncompactness effects, its basic assumption of reactor independence is questionable. For example, flamelet studies show that disturbances generated at one point of the flame convect downstream and affect its dynamics at other points. However, as is the more general problem with these reactor-based approaches, it is not clear how to incorporate these interaction effects in a rational manner.

None of these reactor-based models has been subjected to direct experimental verification. (By direct verification, we preclude qualitative comparisons between experiments and a larger stability model within which the reactor serves as a submodel.) As such, the accuracy of these models is questionable. Furthermore, the heuristic nature of reactor models makes it unlikely that they can be used for quantitative predictions of combustion process response to acoustic perturbations. Two major conceptual issues associated with reactor models should be emphasized. First, reactor models are known to be useful for correlating the steady-state blowout characteristics of high-intensity flames.[52] It is likely that the intense recirculation regions that stabilize these flames have distributed reactorlike properties, hence, the success in reactor models in correlating blowout behavior. However, in many cases, it is also possible that the majority of the flame has flamelet-like properties. In these situations, a model that is only valid in a small region of the combustion process will not accurately describe its overall dynamics; that is, unsteady reactor models do not necessarily describe the overall dynamics of a combustion region, even if its blowout limits can be satisfactorily correlated with a steady state prediction.

Second, as already alluded to, it will be very difficult to model rationally the interactions between separate reactorlike regions in space that see different disturbance values (such as mixture composition), as well as the response of the reactor residence time to perturbations. This second point seems particularly severe, as can be illustrated by the following points. Consider two identical reactors fed by the same fuel flow rates, but at different pressures, p_1 and p_2. Under the assumption that the all of the fuel is reacted to form products, it is clear that the total heat release of both reactors is also the same. Now, assume that the pressure in either reactor oscillates in time between the two values, that is, $p(t) = p_1 + (p_2 - p_1)\sin \omega t$. Under the assumption that the frequency is low enough, the reactor will respond in a quasi-steady manner, implying that the total rate of heat release remains constant. A similar argument can be made for a reactor disturbed by other fluctuations, such as temperature. Only fluctuations in the heat content of the inlet fuel stream will cause a quasi-steady fluctuation in reactor heat release. What is happening? Clearly, the changes in pressure or temperature influence the volumetric reaction rate, for example, heat release/volume. However, in the quasi-steady case, increases in reaction rate are accompanied by reductions in overall reaction volume, that is, the same amount of heat is released, but over a smaller volume. This discussion shows that quasi-steady perturbations that do not affect the heating content of the inlet stream do not introduce fluctuations in overall heat release. Thus, any fluctuation in heat release that does occur is a dynamic effect, that is, the pressure perturbations referred to could potentially cause heat release oscillations at sufficiently large frequency ω. In this case, it is then necessary to model the dynamics of the global reaction zone response to the perturbation. We will show in Sec. IV that an analogous situation occurs in the flamelet regime, where quasi-steady fluctuations in flame speed do not cause the global heat release to oscillate because of the accompanying oscillations in flame area. The difference is that these coupled dynamics can be reasonably modeled from first principles in the flamelet case; how this would be done in the reactor case is uncertain.

To conclude, although reactor models are attractive due to their simplicity, they are not currently useful as a quantitative predictive tool because the relevant dynamic interactions cannot be modeled in a rational manner. As such, for the foreseeable future their utility will at best be restricted to semi-empirical correlations.

B. Realistic Geometries: Linear Response

This section considers the interaction of flames in more realistic geometries, such as ducted flames stabilized at rapid expansions or bluff bodies. The principle distinction between this problem and those already considered is the introduction of a length scale associated with the physical size of the combustion region.

Numerous phenomenological models of the flame transfer function have been reported, for example, Becker and Gunther,[101] Merk,[102] or Matsui.[103] Marble and Candel[104] appear to have performed the first physics-based calculation of the flame sheet's dynamic response to acoustic perturbations. Their work was followed with similar studies by Subbaiah[105] and Poinsot et al.[106] and incorporated into a combustion instability model by Yang and Culick.[107] The geometry considered by Yang and Culick is shown in Fig. 12. These investigations all considered the interaction of a flame stabilized in a combustor where the flame was inclined to the flow so that the resultant flowfield was two dimensional.

Marble and Candel's[104] analysis was motivated by the fact that self-excited combustor oscillations in many systems, such as large boilers or aircraft engine afterburners, occur at low frequencies where chemical kinetic processes likely exert minimal influence on the dynamics of the interaction. They note, however, that distortions of the overall flame geometry, such as can be seen in Fig. 3, are propagated along the front by the mean flow speed, that is, the flame's response depends primarily on fluid mechanic adjustments. Thus, the relaxation time of the overall flame front occurs on a time-scale given by the ratio of the flame length and mean flow speed.

The essential approach used in this study[104] is similar in most respects to the flamelet studies described in the preceding section. The flame dynamics are described by the kinematic equations given in Eqs. (1–3). Acoustic oscillations, assumed to be one dimensional, are matched using the jump conditions, such as in Eq. (4–7). Free-boundary problems such as these are extremely difficult to handle analytically. To proceed, these studies[104–107] used an integral technique where the equations were integrated in the transverse direction between the flame and combustor wall. Calculations of the acoustic pressure reflection and transmission coefficient showed well-defined peaks at certain values of the flame Strouhal number, defined earlier as $Sr = f L_{\mathrm{FL}}/u$. Because the quantity L_{FL}/u refers to the amount of time required for a disturbance to convect along the flame at the flow speed, these results showed that the flame amplified disturbances with characteristic times $T = 1/f$ that matched particular multiples of this characteristic convective time. Thus, these studies clearly showed the importance of considering a finite sized flame region in accounting for the interactions between acoustic disturbances and the flame front. The significance of this Strouhal number has been well validated in several experimental studies of the flame transfer function, for example, see Refs. 57 and 108.

Further progress has been made in several more recent studies using similar approaches by Boyer and Quinard,[109] Baillot et al.,[110] Fleifel et al.,[111] and Ducruix et al.[57] These studies circumvented the analytical difficulties encountered in the described analyses by neglecting the coupling of flow perturbations across the flame. Rather, they calculated the response of the flame from an imposed velocity disturbance of given amplitude and phase upstream of the flame. In essence, this approximation neglects the density change across the flame front. Nonetheless, the substantially reduced complexity of their approach facilitates much more transparent analysis; moreover, their results appear to give good agreement with experiments.

The theoretical results of Boyer and Quinard[109] and Baillot et al.[110] focused on prediction of the flame shape and were in good agreement with their experimental results. The analyses of Fleifel et al.[111] and Ducruix et al.[57] focused on prediction of the overall flame front response, characterized by the overall flame front area. Similar to the analyses described earlier, they found that a Strouhal number based on flame length and flow velocity (or, roughly equivalent, the flame speed and duct radius) was the dominant parameter controlling the amplitude and phase of the flame response for a given flame shape. Their analysis will be presented in more detail in Sec. IV. More recent measurements of Ferguson et al.[112] suggest, however, that the flame's heat release response cannot be uniquely characterized by its instantaneous area, as assumed in Refs. 57 and 111. This study[112] found nonnegligible phase differences between instantaneous measurements of flame surface area and OH* chemiluminescence.

The analyses in Refs. 57, 86, 87, and 104–111 show that the flame exhibits a strong response to acoustic velocity perturbations, that is, velocity coupling. These studies all assumed that these local velocity perturbations were one dimensional. Basic acoustic considerations suggest that such a description of the flame's acoustic near field is unrealistic, however, given the multidimensionalities in the geometric and flame front configuration. A planar incident wave impinging on a flame front not only generates planar reflected and transmitted waves, but also multidimensional disturbances that are generally evanescent for the frequencies of interest.[65] Because the flame is disturbed by the local, multidimensional acoustic field, Lee and Lieuwen[113] argued that calculations of its interactions with the acoustic field must account for these multidimensional characteristics. They presented computational results of calculations of the flame's acoustic near field showing that the acoustic velocity could exhibit substantial multidimensional characteristics (e.g., Fig. 13). They then substituted these acoustic field calculations into the flame dynamic calculations and compared the results to those obtained by Fleifel et al.[111] and Ducruix et al.[57] Surprisingly, they found that the two results were qualitatively similar over a wide range of parameters, although they could quantitively differ by factors up to two or three.

Numerous experimental investigations (e.g., Refs. 25, 37, 43, 51, 58–60, and 144) have demonstrated the substantial interactions between premixed flames with intrinsically unstable or acoustically forced coherent vortical structures. Two primary mechanisms of heat release modulation by these structures have received the most

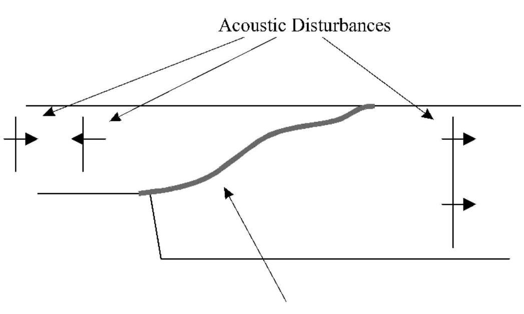

Fig. 12 Interaction of acoustic waves with inclined, ducted flame sheet, such as considered by Yang and Culick.[96]

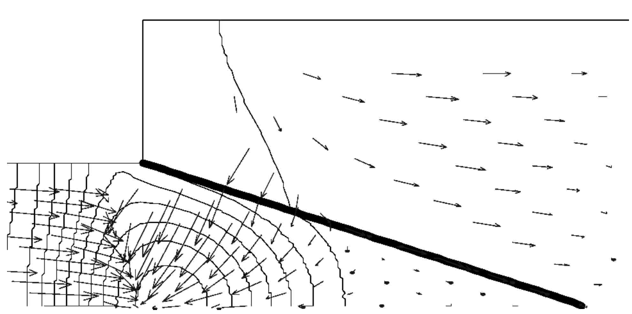

Fig. 13 Dump stabilized combustor geometry for case where the flame was excited from upstream: ——, instantaneous pressure contours and velocity vectors and ▬▬, average flame location (adapted from Lee and Lieuwen[113]).

attention in the literature: flame area modulation[59] or large-scale entrainment of reactive mixture that combusts in a rapid burst.[60] The former interaction is similar in nature to the heat release perturbations induced by acoustic velocity perturbations. Because of the substantial literature that has appeared on this subject alone, we refer the reader to the thorough review papers of Coats[29] and Schadow and Gutmark[28] for a more extensive citation of prior work and discussion of the interaction phenomenology.

To conclude, the kinematics of laminar flame–acoustic wave interactions appears to be well understood, in both the linear and nonlinear (as will be discussed in the next section) case. Excellent agreement has been achieved between surprisingly simple theory and experiments. Because most of the models included purely kinematic effects, this agreement does not imply that the inner dynamics of the reaction zone itself or of the flame holding characteristics are understood: As noted in Sec. III.A.1, the reaction zone dynamics have been subjected to extensive theoretical investigation with few supporting experiments. These points do suggest, however, that the flame response (at least over the investigated parameter space) is dominated by the velocity coupled kinematic response of the flame sheet consistent with the arguments of Clanet et al.[56]

C. Nonlinear Response

The discussion in Secs. III.A and III.B focused attention on the linear response of premixed flames to small-amplitude perturbations. Understanding the nonlinear response of flames to finite amplitude perturbations is critical to predicting limit-cycle amplitudes. Also, in certain situations, it is conceivable that the flame–acoustic problem is inherently nonlinear, for example, in the case where acoustic disturbances are exciting a flame in the presence of background turbulence that has sufficient intensity to cause a nonlinear flame response or even cause local extinction of the flame. Furthermore, recent pressure–heat release transfer function measurements of Lieuwen and Neumeier[115] indicate that the nonlinear dynamics of combustors are dominated by these heat release–acoustic nonlinearities, (Fig. 14).

Consider first these interactions with a region of distributed reaction. The basic unsteady WSR equations (11–15) consist of a system of nonlinear, first-order equations. Thus, incorporating finite amplitude effects into unsteady WSR calculations is straightforward, although it may require numerics for time stepping. The model used by Richards et al.,[96] Rhode et al.,[116] and Lieuwen et al.[98] incorporate these nonlinear equations and, in certain cases, find complex, and even chaotic, combustor dynamics. As in the linear studies, stipulating the manner in which the reactor residence time depends on the other perturbations was not attempted.

Similarly, the finite amplitude response of a flame sheet to acoustic perturbations is described by the nonlinear, partial differential equations given in Eqs. (1–3). In practice, treatment of these equations is made difficult because of the tendency of the flame position to become a multivalued function of the radial or axial coordinate, for pockets to pinch off and form islands and for sharp cusps to appear. This problem has been treated in several analyses, which, though not directly aimed at the acoustic flame interaction problem, did analyze the flame front kinematics in an unsteady and/or periodic flowfield.[117–120] These studies were able to reproduce a number of flame front phenomenon that are observed in experiments such as cusp and pocket formation or flame area amplification.

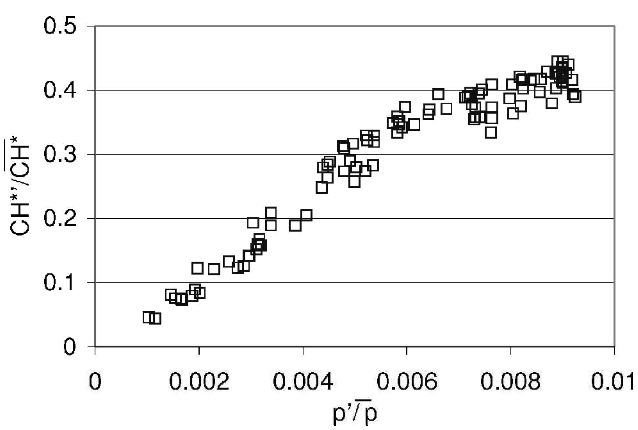

Fig. 14 Measured amplitude relationship between pressure and CH*′ radical chemiluminescence in a premixed combustor.[115]

Bourehela and Baillot,[121] Baillot et al.,[122] and Durox et al.[123] performed a systematic experimental and theoretical study of the response of a Bunsen flame to velocity perturbations of varying amplitude and frequency. Although their principal observations are quite similar to those previously observed by Blackshear,[124] they appear to be the first systematic characterization of the flame response as a function of perturbation amplitude.[121–123] Similar to Fig. 3, they found that, at low frequencies ($f < 200$ Hz) and velocity amplitudes ($u'/\bar{u} < 0.3$), the flame front wrinkles symmetrically about the burner axis due to a convected wave traveling from the burner base to its tip. At higher frequencies, but similar low amplitudes, they observed a phenomenon they refer to as "filtering," where the effect of the oscillations on the flame front is only evident at the flame base and becomes strongly damped at axial locations farther downstream. This behavior does not appear to be due to the low-pass filtering of the flame front to velocity disturbances described, for example, by Ducruix et al.[57] or Saitoh and Otsuka.[125] It may be due to the increased importance of the flame's curvature dependent burning velocity and the very short convective wavelengths of the imposed disturbances at these higher frequencies.

In Refs. 121–123, with an increase of amplitude of low-frequency velocity perturbations, the authors found that the flame exhibited a variety of transient flame holding behavior, such as flashback, asymmetric blowoff, unsteady lifting and reanchoring of the flame. In addition, they note that its response is asymmetric and extremely disordered. Finally, at high frequencies and forcing amplitudes, they found that the flame collapses and no longer has a sharp tip. Rather, the tip region becomes rounded off and, for sufficiently high forcing intensities ($u'/\bar{u} > 1$), the flame's mean shape becomes hemispherical.[121] This latter phenomenon was more extensively investigated by Durox et al.,[123] who showed that the flame's surface area was unmodified when the flame flattens. They note that this flame flattening is equivalent to the phenomenon of acoustic stabilization of cellular flames.

Baillot et al.[122] also reported a theoretical study of these flame fronts to predict their unsteady evolution, particularly at larger amplitudes of forcing, where the flame front becomes strongly cusped. They reduced the flame dynamic equation [Eq. (2)], to a Hamilton–Jacobi type equation, which they solved by the method of characteristics. They found that the predicted and measured flame shapes were in good agreement.

Dowling[126] introduced a phenomenological model for the finite amplitude response of the flame to velocity perturbations. The model is dynamic in nature, but the essential nonlinearity is introduced from a quasi-steady relation between flow velocity and heat release rate. Specifically, it assumes a linear relation between the heat release Q and velocity perturbation when the total velocity ($u = \bar{u} + u'$) lies between 0 and $2\bar{u}$. When $u < 0$, the heat release goes to zero, and when it is greater than $2\bar{u}$ it saturates at $2\bar{Q}$. This relationship is shown in Fig. 15.

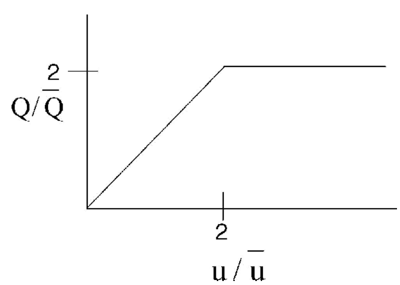

Fig. 15 Relation between quasi-steady heat release and velocity perturbation used by Dowling.[126]

In a later study, Dowling[127] presented a more physics-based model, based on the flame kinematics approaches described earlier (e.g., Refs. 57, 104, 109, and 111. To treat finite amplitude oscillations, Dowling utilized a nonlinear boundary condition at the flame anchoring point. Specifically, she assumed that the flame remained anchored at the bluff body if the total gas velocity exceeded the flame speed. If the gas velocity fell below the flame speed, the former condition was replaced by one that allowed the flame to propagate upstream. Dowling notes that the predicted flame behavior is consistent with direct visual observations from prior experiments.

Peracchio and Proscia[128] developed a quasi-steady nonlinear model to describe the response of the flame to equivalence ratio perturbations. First, they assumed the following relationship for the response of the instantaneous mixture composition leaving the nozzle exit to velocity perturbations:

$$\phi(t) = \bar{\phi}/[1 + ku'(t)/\bar{u}] \tag{24}$$

where k is a constant with a value near unity. They also utilized a nonlinear relationship relating the heat release per unit mass of mixture to the instantaneous equivalence ratio.

To conclude, note that investigations of nonlinear combustion dynamics have been primarily theoretical. The reactor-based models suffer the same shortcomings already noted in Sec. III.A.2. Their utility is primarily restricted toward serving as interesting model problems to study complex, chaotic dynamics. In the flamelet case, good agreement has been achieved between theory and experiment for simple laminar flames. As already noted, this agreement suggests that the linear and nonlinear kinematics of flame's responding to acoustic disturbances is well understood. Other analyses of nonlinearity mechanisms, such as flame holding, the equivalence ratio–velocity relationship, the equivalence ratio–heating value relationship, and others, seem reasonable but have only been compared qualitatively with measurements.

IV. Flame Transfer Function Calculations

Many of the concepts and conclusions discussed can be reinforced through analysis of a model problem that explicitly allows for calculation of the flame response to perturbations. In this section, we present a model for the kinematic response of a flame sheet to two types of perturbations: velocity and fuel/air ratio. This analysis closely follows that of Fleifel et al.[111] and Ducruix et al.[57] for the velocity perturbation and Cho and Lieuwen[129] for the fuel/air ratio perturbation. This analysis specifically focuses on the global heat release response and is, thus, most relevant to acoustically compact flames (Sec. II.D).

The studied geometry is axisymmetric and shown in Fig. 16. The flame's instantaneous axial position x is given by the following function of the radial and temporal coordinates, r and t:

$$x = \xi(r, t) \tag{25}$$

Thus, we define the flame position surface $f(\boldsymbol{x}, t)$ [Eqs. (1–3)], as $f(\boldsymbol{x}, t) = x - \xi(r, t) = 0$. Note that this definition requires the flame's axial position x to be a single-valued function of the radial coordinate. Substituting this expression into Eq. (2) yields the following kinematic relation describing the flame position as a function of the flow velocity and flame speed:

$$\frac{\partial \xi}{\partial t} = u - v\frac{\partial \xi}{\partial r} - S_1\sqrt{\left(\frac{\partial \xi}{\partial r}\right)^2 + 1} \tag{26}$$

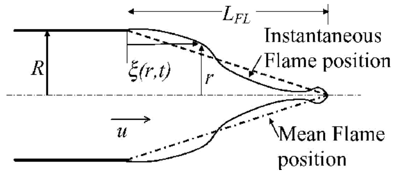

Fig. 16 Schematic of model problem considered in Sec. IV.

where u, v, and S_1 are axial velocity, radial velocity, and flame speed relative to the unburned gases, respectively. We assume that the mean velocity is uniform and purely axial, that is, $\bar{v} = 0$, and that the mean flame speed is constant. Although these assumptions are not necessary to proceed with the analysis, they do yield more transparent results that retain many of the basic phenomena of interest. We next decompose the variables into their mean and fluctuating parts and retain only linear terms in perturbations. This procedure yields the following equations for the mean and fluctuating variables:

$$\bar{u} = \bar{S}_1\sqrt{1 + \left(\frac{\mathrm{d}\bar{\xi}}{\mathrm{d}r}\right)^2} \tag{27}$$

$$\frac{\partial \xi'}{\partial t} = u' - v'\frac{\mathrm{d}\bar{\xi}}{\mathrm{d}r} - \bar{S}_1\left[\frac{(\mathrm{d}\bar{\xi}/\mathrm{d}r)(\partial\xi'/\partial r)}{\sqrt{1 + (\mathrm{d}\bar{\xi}/\mathrm{d}r)^2}}\right] - S_1'\sqrt{1 + \left(\frac{\mathrm{d}\bar{\xi}}{\mathrm{d}r}\right)^2} \tag{28}$$

As in Ref. 111, we simplify Eq. (28) with the additional assumption that $\bar{u} \gg \bar{S}_1$, which implies that the flame length is much greater than its diameter, $L_{\mathrm{FL}} \gg R$, and that

$$\sqrt{1 + \left(\frac{\mathrm{d}\bar{\xi}}{\mathrm{d}r}\right)^2} \approx -\frac{\mathrm{d}\bar{\xi}}{\mathrm{d}r}$$

As such, the mean flame position is given by

$$\bar{\xi}(r)/L_{\mathrm{FL}} = 1 - (r/R) \tag{29}$$

Note that $L_{\mathrm{FL}}/R \approx \bar{u}/\bar{S}_1$.

Assuming harmonic oscillations at an angular frequency ω [$\exp(-i\omega t)$] Eq. (28) becomes an ordinary differential equation that can be solved for the flame position. When it is assumed that the flame remains anchored at its base, that is, $\xi'(r = R, \omega) = 0$, the solution takes the following form:

$$\frac{\xi'}{L_{\mathrm{FL}}} = \exp(-iSrr/R)\int_1^{r/R}\left[\left(\frac{S_1'(\eta)}{\bar{S}_1} - \frac{u_n'(\eta)}{\bar{S}_1}\right)\exp(iSr\eta)\right]\mathrm{d}\eta \tag{30}$$

where the Strouhal number definition is slightly different than used before, $Sr = \omega L_{\mathrm{FL}}/\bar{u}$, and u_n' is the unsteady velocity component normal to the mean flame position. Within the assumptions of this analysis, this expression shows that a perturbation in normal velocity or flame speed exerts an identical effect on the flame's position. As will be shown, however, the overall effects of these perturbations on the heat release are different because the flame speed perturbation effects both the flame front area and the local consumption rate, whereas the velocity perturbation only effects the former.

Further progress in analyzing Eq. (30) requires specifying the spatial distribution of the velocity and flame speed perturbation. We assume that the spatial distribution of these two perturbations are acoustic and convective in nature, to illustrate the effects that the different disturbances modes have on the flame's response. As such, u_n' is assumed to be spatially uniform, a reasonable assumption if the flame is acoustically compact. The flame speed perturbation is assumed to be generated by a convected perturbation in mixture stoichiometry, given by

$$S_1' = \left(\frac{\mathrm{d}S_1}{\mathrm{d}\phi}\right)_{\bar{\phi}} \cdot \phi' \tag{31}$$

where ϕ' is convected by the mean flow and, thus, has an axial distribution given by

$$\phi'(x, t) = \phi_0'\exp[-i\omega(t - x/\bar{u})] = \phi_0'\exp(-i\omega t)\exp[iSr(1 - r/R)] \tag{32}$$

Following Fleifil et al.[111] and Ducruix et al.,[57] the global heat release rate of the flame is written as

$$Q(t) = \int_S \rho S_1 \Delta h_R \, \mathrm{d}A_{\mathrm{FL}} \tag{33}$$

where the integral is performed over the flame surface, ρ is the density, and Δh_r is the heat of reaction. We assume that the density of the reactive mixture is constant. Fluctuations in the remaining quantities contribute to heat release oscillations as

$$\frac{Q'}{\bar{Q}} = \frac{\int \Delta h_R' \mathrm{d}\bar{A}_{\mathrm{FL}}}{\int \Delta \bar{h}_R \mathrm{d}\bar{A}_{\mathrm{FL}}} + \frac{\int S_1' \mathrm{d}\bar{A}_{\mathrm{FL}}}{\int \bar{S}_1 \mathrm{d}\bar{A}_{\mathrm{FL}}} + \frac{A_{\mathrm{FL}}'}{\bar{A}_{\mathrm{FL}}} \tag{34}$$

where A_{FL} is the flame surface area whose instantaneous value is given by

$$A_{\mathrm{FL}} = \int_0^R 2\pi r \sqrt{1 + \left(\frac{\partial \xi}{\partial r}\right)^2} \, \mathrm{d}r \tag{35}$$

Decomposing A_{FL} into its mean and fluctuating components yields the following linearized result for the area fluctuation term:

$$\frac{A_{\mathrm{FL}}'}{\bar{A}_{\mathrm{FL}}} = 2 \int_0^1 \exp(-iSrr) \left[\int_r^1 \left(\frac{u_n'(\eta)}{\bar{S}_1} - \frac{S_1'(\eta)}{\bar{S}_1} \right) \exp(iSr\eta) \, \mathrm{d}\eta \right] \mathrm{d}r \tag{36}$$

Pulling the results from Eqs. (34) and (36) together, we arrive at the following expression that decomposes the heat release perturbation into its contributions from the velocity and equivalence ratio perturbation:

$$Q'/\bar{Q} = Q'/\bar{Q}|_{u'} + Q'/\bar{Q}|_{\phi'} \tag{37}$$

Define the following flame transfer functions to perturbations in velocity, F_u, and equivalence ratio, F_ϕ:

$$F_u = \frac{Q'/\bar{Q}|_{u'}}{u_n'/\bar{S}_1} = \frac{2}{Sr^2}[1 + iSr - \exp(iSr)] \tag{38}$$

$$F_\phi = \frac{Q'/\bar{Q}|_{\phi'}}{\phi_b'/\bar{\phi}} = F_H + F_S = F_H + (F_{S,\mathrm{dir}} + F_A) \tag{39}$$

where

$$F_H = \left.\frac{\mathrm{d}(\Delta h_R/\Delta \bar{h}_R)}{\mathrm{d}(\phi/\bar{\phi})}\right|_{\bar{\phi}} \frac{2}{Sr^2}\{1 + iSr - \exp(iSr)\}$$

$$F_{S,\mathrm{dir}} = \left.\frac{\mathrm{d}(S_1/\bar{S}_1)}{\mathrm{d}(\phi/\bar{\phi})}\right|_{\bar{\phi}} \frac{2}{Sr^2}\{1 + iSr - \exp(iSr)\}$$

$$F_A = \left.\frac{\mathrm{d}(S_1/\bar{S}_1)}{\mathrm{d}(\phi/\bar{\phi})}\right|_{\bar{\phi}} \frac{2}{Sr^2}\{1 - (1 - iSr)\exp(iSr)\}$$

and ϕ_b' in Eq. (39) is the perturbation in equivalence ratio at the flame base. Note that these transfer functions are solely a function of the Strouhal number and the sensitivities of the heat of reaction and flame speed to the equivalence ratio. The amplitude and phase dependence of these transfer functions [Eqs. (38) and (39)] are shown in Fig. 17. These characteristics are described in more detail later.

The equivalence ratio transfer function F_ϕ has three contributing terms [Eq. (39)]. The first term, F_H, is due to perturbations in heat of reaction, that is, the heat content of the reactive mixture. The second term, F_S, is due to perturbations in flame speed. Note that perturbations in flame speed are again divided into two factors; one is directly generated by the flame speed sensitivity to equivalence ratio, $F_{S,\mathrm{dir}}$, and the other is due to the subsequent fluctuation in flame surface area, F_A. Our explicit calculations assume a quasi-steady relationship between equivalence ratio and flame speed, that is, that $\mathrm{d}(S_l/\bar{S}_l)/\mathrm{d}(\phi/\bar{\phi})$ is independent of frequency. Incorporating the additional dynamics of this relationship can be added in a straightforward manner if necessary, however.

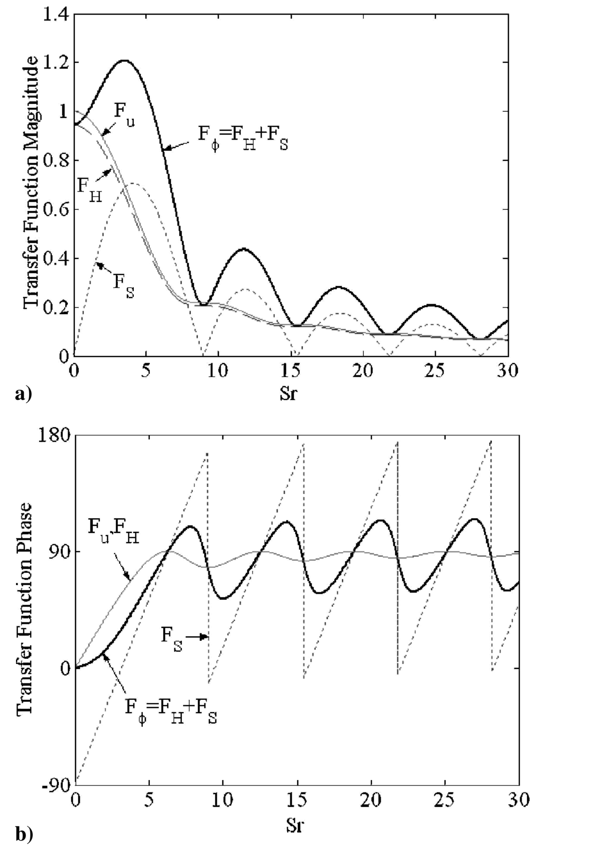

Fig. 17 Amplitude dependence of flame's heat release response to flame speed and acoustic velocity oscillations on Strouhal number.

In general, the relationship between unsteady heat release rate and the velocity or equivalence ratio perturbation has a complex dynamic. However, for $Sr \ll 1$, that is, a convectively compact flame, the Q_u' relationship can be put in terms of a simple n–τ model,

$$Q_u'(t)/\bar{Q} = n_u u_n'(t - \tau_u) \tag{40}$$

where $n_u = 1/\bar{S}_1$ and $\tau_u = L_{\mathrm{FL}}/3\bar{u}$.

The dynamics of Q_ϕ' cannot be generally described by an n–τ model, even in the $Sr \ll 1$ limit. This is due to the possible negative phase dependence of F_ϕ on Strouhal number when $Sr \ll 1$ (Fig. 18) that is, the flame can not respond before the equivalence ratio perturbation reaches it. The low Sr dynamics of Q_ϕ' is given by

$$\frac{Q_u'(t)}{\bar{Q}} = n_H \phi_b'(t - \tau_H) + n_S \frac{\mathrm{d}\phi_b'(t)}{\mathrm{d}t} \tag{41}$$

where

$$n_H = \left.\frac{\mathrm{d}(\Delta h_R/\Delta \bar{h}_R)}{\mathrm{d}\phi}\right|_{\bar{\phi}}, \qquad \tau_H = \frac{L_F}{3\bar{u}}, \qquad n_S = \frac{1}{3}\frac{L_F}{\bar{u}} \left.\frac{\mathrm{d}(S_1/\bar{S}_1)}{\mathrm{d}\phi}\right|_{\bar{\phi}}$$

Equation (40) indicates that the time response of the heat release rate to perturbations in acoustic velocity, $Q_u'(t)$, is delayed by a retarded time, τ_u, where τ_u represents the time taken for the mean flow to convect a distance of one-third of the flame length, which can be taken as the effective position of concentrated heat release, that is, $L_{\mathrm{eff}} \approx L_F/3$. However, the heat release response to equivalence ratio perturbations, $Q_\phi'(t)$, is delayed or advanced depending on the combined effect of τ_H and a temporal rate of change of flame speed perturbations as shown in Eq. (41).

To quantify the dependence of heat of reaction and flame speed on equivalence ratio, we use the following correlation from Abu-Off and Cant[130] for methane at 300 K and atmospheric pressure (also used by Hubbard and Dowling[131] for a similar calculation):

$$S_1(\phi) = A\phi^B e^{-C(\phi - D)^2} \tag{42}$$

$$\Delta h_R(\phi) = \frac{2.9125 \times 10^6 \,\mathrm{minimum}(1, \phi)}{1 + \phi 0.05825} \tag{43}$$

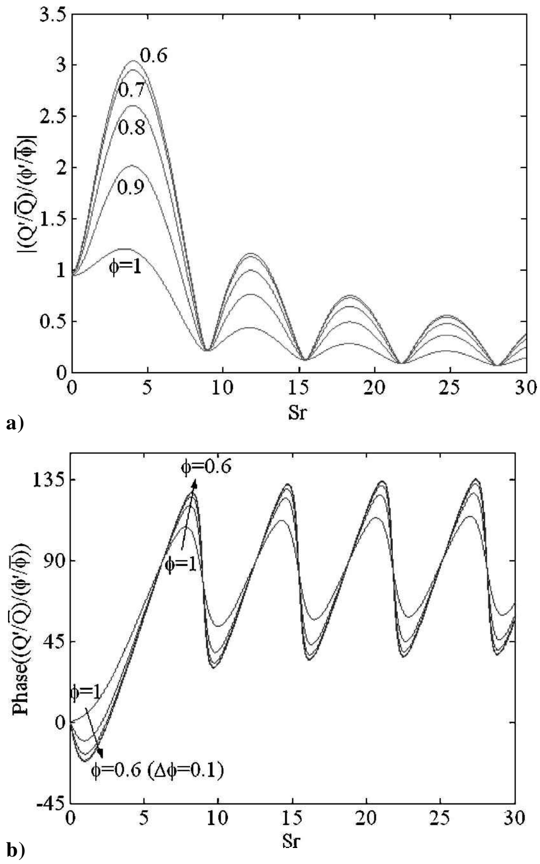

Fig. 18 Phase dependence of flame's heat release response to flame speed and acoustic velocity oscillations upon Strouhal number.

where the following values were used for the constants: $A = 0.6079$, $B = -2.554$, $C = 7.31$, and $D = 1.230$.

These correlations were used to generate the results in Figs. 17–19. Refer to Figs. 17 and 19, and note that F_u and F_H are nearly identical and decrease monotonically from their maximum response at $Sr = 0$. In contrast, the heat release response to flame speed perturbation, F_S, vanishes at $Sr = 0$. This is due to the exact cancellation of the flame speed and area perturbation terms, which have equal magnitudes, but opposite phases. This zero response at $Sr = 0$ can be understood from quasi-steady arguments, that is, the flame area fluctuates with the same magnitude and opposite phase as the flame speed oscillations. This zero response can also be understood from the fact that the flame speed and area perturbation terms account for the flame's response to a mixture with constant heat of reaction. For example, two substoichiometric flames with the same flow of fuel, but differing amounts of air, release the same amount of heat for quasi-steady states, although the flame's have different areas. As such, slow timescale perturbations may affect the flame's local consumption rate, but the resultant heat release perturbation is exactly balanced by the resultant variations in flame area.

The latter transfer function F_S increases with Strouhal number from zero because of changes in the relative phase of the terms $F_{S,\mathrm{dir}}$ and F_A. It reaches a global maxima at $Sr \sim 4.5$, where the two perturbations reinforce each other. As the Strouhal number increases further, F_S decreases in an oscillatory pattern due to the alternating phase relationship between $F_{S,\mathrm{dir}}$ and F_A. The total heat release response, F_ϕ, increases until $Sr \sim 4$ and decreases in an oscillatory manner.

Figure 17b shows the dependence of the phase of the transfer functions on Strouhal number. As in Eqs. (38) and (39), the acoustic velocity term F_u and the heat of reaction term F_H have exactly the same Strouhal number dependence. They start at 0 deg at $Sr = 0$ and increase monotonically to and oscillate around 90 deg. On the other hand, the flame speed term F_S starts at a phase of -90 deg. This is the average of $F_{S,\mathrm{dir}}$ and F_A, which start at 0 and -180 deg, respectively. The phase of F_ϕ lies between F_H and F_S. This phase of

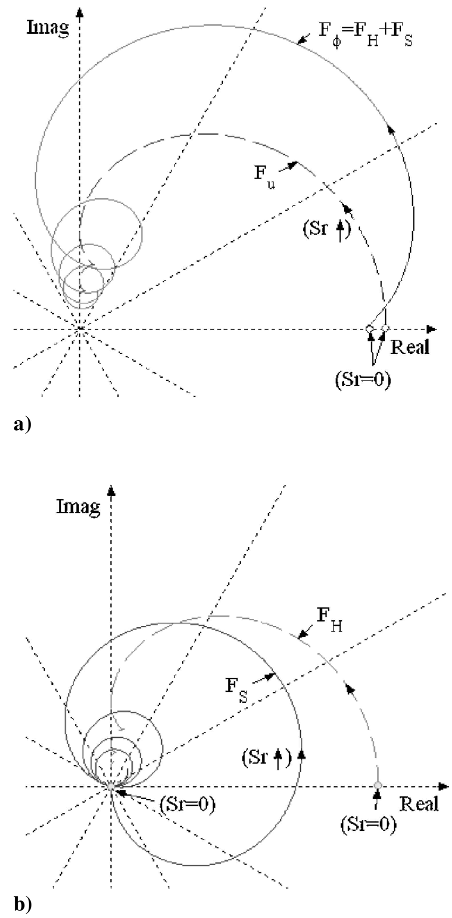

Fig. 19 Polar plot of flame transfer functions, where Strouhal number *Sr* is a parameter, $\phi = 1$.

F_ϕ initially increases slightly and then increases more rapidly along with the phase of F_S toward 90 deg.

Figure 18 shows the effect of the mean equivalence ratio on the flame transfer function using the correlations in Eqs. (42) and (43). It shows that mixture stoichiometry exhibits little effect on the transfer function magnitude for $Sr \ll 1$ and at subsequent minima. In most cases, however, the flame response increases with decreasing equivalence ratio. This is due to the increased sensitivity of the flame speed to equivalence ratio for lean mixtures. Note that the Strouhal number value of maximum response, $Sr \sim 5$, remains essentially constant. Figure 18b shows the phase-dependence on Strouhal number at several equivalence ratios. Note that the low Strouhal number heat release response can either lead or lag the ϕ perturbation, depending on mean ϕ value.

The predicted amplitude and phase characteristics of the velocity transfer function F_u have been shown by Ducruix et al.[57] to agree well with experiments up to $\mathrm{Sr} \sim 8$ (Substantial phase deviations between theory and experiment were observed for larger Strouhal numbers.) This good agreement provides confidence in the basic kinematic modeling approach. Note that no measurements of the equivalence ratio transfer function F_ϕ have been made.

An important conclusion to be drawn from these analyses is the importance of both the local and global effect of a perturbation on the overall flame response. For example, a flame speed perturbation causes both a local change in heat release rate/unit area, as well as in the overall flame area. The transfer function results illustrated in Fig. 17 show that inclusion of both effects is crucial in modeling the overall flame response.

This comparison also illustrates the difficulties associated with using unsteady WSRs to model flame–acoustic interactions. For example, consider the Lieuwen et al.[62] analysis of the response of a WSR to equivalence ratio perturbations, where the WSR volume was assumed fixed. Analogous to the area effect in the preceding flamelet calculation, this assumption of fixed reactor volume likely causes their results to be erroneous, that is, increases in the mixture's local reaction rate result in reductions in reaction region volume. At low enough frequencies, these two effects (reaction rate and volume) must cancel.

V. Conclusions

From a combustor designers viewpoint, the goal of this work is to develop a model that can rapidly predict the qualitative, and preferably quantitative, dependence of combustor stability on geometric and fuel composition parameters. Accurately modeling the combustion process response to flow perturbations is a critical component of such a capability. Reasonable, quantitative predictions of flame–acoustic interaction phenomenon has been demonstrated for a few simple configurations, such as the laminar Bunsen burner of Ducruix et al.[57] or Baillot et al.,[110] or the nominally flat flame of Searby and Rochwerger.[86] These successes are illustrative of the rapid progress being made in modeling kinematic processes in flame–acoustic interactions. In addition, progress is being made in developing hybrid models that use computational simulations to determine various components of the combustor system–flame interactions.[132] The development of accurate, predictive combustion response models for realistic, that is, turbulent, configurations has not been achieved, however, and remains a key challenge for future workers. The discussion below suggests some requirements needed to achieve this capability.

First and most generally, it seems critical that better coordination between models and experiments be achieved. At present, a significant part of the relevant literature consists of essentially decoupled theoretical models or experimental studies, even in rather fundamental configurations. For example, a substantial number of fundamental studies have theoretically investigated the response of flat, laminar flames to pressure perturbations.[71–77] No serious effort appears to have been initiated to subject these predictions to experimental scrutiny. Similarly, although equivalence ratio oscillations are known to play an important role in driving instabilities, no experimental work appears has been performed to examine the accuracy of models that relate them to the resultant heat release oscillations. Even though these highly fundamental studies may be far removed from practical flames, they are prerequisite building blocks toward modeling realistic systems.

Second, work is needed to develop simplified models of vortex–flame interactions. The existing theoretical work on this subject is largely numeric. Analytic methodologies for modeling unstable, reacting shear flows have been developed[133] and need to be extended to incorporate the unsteady flow effects on the flame.

Third, predicting the response of flames to finite amplitude waves is immature. The few existing studies are largely phenomenological in nature. For example, it is not presently clear whether limit-cycle amplitudes in lean, premixed gas turbines are controlled by nonlinear flame front dynamics, the nonlinear response of the equivalence ratio to velocity perturbations in the premixer, chemical kinetic effects, or some other process. Substantial progress could be made by a set of experiments that isolate the key nonlinear processes that need to be focused upon by modelers. Interpretive guidance of these results can be achieved by parallel systematic studies of potential nonlinearities. In addition, effects such as the stabilization or parametric destabilization of flames discussed in Sec. III.A may cause finite amplitude acoustic oscillations to change substantially the characteristics of the turbulent flame it is interacting with. These effects merit further investigation.

Fourth, flame–acoustic wave interactions in realistic environments occur in a very noisy atmosphere, where the flame is a highly perturbed front, even in the absence of coherent acoustic oscillations, and executes large oscillations about its mean position. Any model of the response of laminar flame fronts to velocity, equivalence ratio, or vortical disturbances needs to be generalized to include the fact that the "average" flame is highly unsteady. For example, the successful work performed to date on laminar, Bunsen flames should be extended to turbulent flow situations. Fundamental issues, such as how the transfer functions measured by Ducruix et al.[57] change with increasing turbulence levels, need to be addressed.

Fifth, the interactions of flames with thickened flamelets, distributed reaction zones, or well-stirred reactions needs to be considered. As already emphasized, current WSR models are largely phenomenological and have a number of significant conceptual problems. Progress in this area will require clarification of the nature of the combustion process in this regime by the turbulent combustion community.

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References

[1]Putnam, A., *Combustion Driven Oscillations in Industry*, American Elsevier Publishers, New York, 1971.

[2]Price, E. W., "Combustion Instability in Solid Propellant Rocket Motors," *Astronautica Acta*, Vol. 5, 1959.

[3]Crocco, L., and Cheng, S., *Theory of Combustion Instability in Liquid Propellant Rocket Motors*, Butterworths Scientific, London, 1956.

[4]Hegde, U. G., Reuter, D., Daniel, B. R., Zinn, B. T., "Flame Driving of Longitudinal Instabilities in Dump Type Ramjet Combustors," *Combustion Science and Technology*, Vol. 55, 1987, pp. 125–138.

[5]Keller, J. J., "Thermoacoustic Oscillations in Combustion Chambers of Gas Turbines," *AIAA Journal*, Vol. 33, No. 12, 1995, pp. 2280–2287.

[6]Richards, G. A., and Janus, M. C., "Characterization of Oscillations During Premix Gas Turbine Combustion," American Society of Mechanical Engineers, ASME Paper 97-GT-244, 1997.

[7]Shih, W. P., Lee, J., and Santavicca, D., "Stability and Emissions Characteristics of a Lean Premixed Gas Turbine Combustor," *Proceeding of the Combustion Institute*, Vol. 26, 1996, pp. 2771–2778.

[8]Kendrick, D. W., Anderson, T. J., and Sowa, W. A., "Acoustic Sensitivities of Lean-Premixed Fuel Injectors in a Single Nozzle Rig," American Society of Mechanical Engineers, ASME Paper 98-GT-382, 1998.

[9]Anderson, T. J., and Morford, S. A., "Dynamic Flame Structure in a Low NOx Premixed Combustor," American Society of Mechanical Engineers, ASME Paper 98-GT-568, 1998.

[10]Lawn, C. J., "Thermo-acoustic Frequency Selection by Swirled Premixed Flames," *Proceedings of the Combustion Institute*, Vol. 28, 2000, pp. 823–830.

[11]Lawn, C. J., "Thermo-acoustic Response of a Premixed Swirl Burner," *Proceedings of the Institute of Mechanical Engineers, Part A*, Vol. 214, No. 4, 2000, pp. 333–354.

[12]Paschereit, C. O., and Polifke, W., "Investigation of the Thermoacoustic Characteristics of a Lean Premixed Gas Turbine Burner," American Society of Mechanical Engineers, ASME Paper 98-GT-582, 1998.

[13]Milosavljevic, V. D., Sivasegaram, S., and Whitelaw, J. H., "Velocity Characteristics of Oscillating Ducted Premixed Flames," *Experiments in Fluids*, Vol. 9, 1990, pp. 111–115.

[14]Rayleigh, J. S. W., *The Theory of Sound*, Vol. 2, New York, 1945.

[15]Yu, K., Trouve, A., and Daily, J., "Low-Frequency Pressure Oscillations in a Model Ramjet Combustor," *Journal of Fluid Mechanics*, Vol. 232, 1991, pp. 47–72.

[16]Zinn, B. T., "Pulsating Combustion," *Advanced Combustion Methods* edited by F. Weinberg, Academic Press, New York, 1986.

[17]Keller, J. O., Bramlette, T., Westbrook, C., and Dec, J., "Pulse Combustion: The Quantification of Characteristic Times," *Combustion and Flame*, Vol. 79, 1990, pp. 151–161.

[18]Dowling, A., and Stow, S., "Linear Waves in Gas Turbine Combustors," *Journal and Propulsion and Power*, Vol. 19, No. 5, 2003, pp. 751–764.

[19]Munjal, M., *Acoustics of Ducts and Mufflers*, Wiley, New York, 1987.

[20]Culick, F. E. C., "Nonlinear Growth and Limiting Amplitude of Acoustic Oscillations in Combustion Chambers," *Combustion Science and Technology*, Vol. 3, No. 1, 1971.

[21]Bloxsidge, G., Dowling, A., and Langhorne, P., "Reheat Buzz: An Acoustically Coupled Instability. Part 2. Theory," *Journal of Fluid Mechanics*, Vol. 193, 1988, pp. 445–473.

[22]De Luca, L., Price, E., and Summerfield, M., *Nonsteady Burning and Combustion Stability of Solid Propellants*, Progress in Aeronautics and Astronautics, Vol. 143, AIAA, Washington, DC 1992.

[23]Tong, A., and Sirignano, W., "Multicomponent Transient Droplet Vaporization: Integral Equation Formulation and Approximate Solution," *Numerical Heat Transfer*, Vol. 10, 1986, pp. 253–278.

[24]Harrje, D., and Reardon, F. (ed.), *Liquid Propellant Rocket Instability*, NASA SP-194, 1972.

[25]Broda, J. C., Seo, S., Santoro, R. J., Shirhattikar, G., and Yang, V., "An Experimental Investigation of Combustion Dynamics of a Lean, Premixed Swirl Injector," *Proceedings of the Combustion Institute*, Vol. 27, 1998, pp. 1849–1856.

[26]Hertzberg, J., Shepherd, I., and Talbot, I., "Vortex Shedding Behind Rod Stabilized Flames," *Combustion and Flame*, Vol. 86, No. 1–2, 1991, pp. 1–11.

[27]Yu, K. H., Trouve, A., and Candel, S., "Combustion Enhancement of a Premixed Flame by Acoustic Forcing with Emphasis on Role of Large Scale Vortical Structures," AIAA Paper 91-0367, 1991.

[28]Schadow, K., and Gutmark, E., "Combustion Instability Related to Vortex Shedding in Dump Combustors and Their Passive Control," *Progress in Energy and Combustion Science*, Vol. 18, 1992, pp. 117–132.

[29]Coats, C., "Coherent Structures in Combustion," *Progress in Energy and Combustion Science*, Vol. 22, 1996, pp. 427–509.

[30]Reynst, F., *Pulsating Combustion: The Collected Works of F.H. Reynst*, edited by M. Thring, Pergamom, New York, 1961.

[31]Yang, V., and Anderson, W. (ed.), *Liquid Rocket Engine Combustion Instability*, Progress in Astronautics and Aeronautics, Vol. 169, AIAA Publications: Washington, DC, 1995.

[32]Natanzon, M. S., *Combustion Instability*, edited by F. Culick, Mashinostroyeniye, Moscow, 1999 (translated electronically, 1996).

[33]Culick, F., Heitor, M. V., and Whitelaw, J. H. (eds.), *Unsteady Combustion*, Kluwer, Boston, 1996.

[34]Markstein, G. H., *Nonsteady Flame Propagation*, Pergamon, New York, 1964.

[35]Williams, F., *Combustion Theory*, Addison Wesley, Redwood City, CA, 1985.

[36]Oran, E., and Gardner, J., "Chemical-Acoustic Interactions in Combustion Systems," *Progress in Energy and Combustion Science*, Vol. 11, 1985, pp. 253–276.

[37]McManus, K., Poinsot, T., and Candel, S. M., "A Review of Active Control of Combustion Instabilities," *Progress in Energy and Combustion Science*, Vol. 19, 1993, pp. 1–29.

[38]Candel, S., "Combustion Instabilities Coupled by Pressure Waves and their Active Control," *Proceedings of the Combustion Institute*, Vol. 24, 1992, pp. 1277–1296.

[39]Candel, S., "Combustion Dynamics and Control: Progress and Challenges," *Proceedings of the Combustion Institute*, Vol. 29, 2002.

[40]Culick, F., "Combustion Instabilities in Liquid-Fueled Propulsion Systems–An Overview," AGARD, 1977.

[41]Clavin, P., "Dynamics of Combustion Fronts in Premixed Gases: From Flames to Detonations," *Proceedings of the Combustion Institute*, Vol. 28, 2000, pp. 569–585.

[42]Peters, N., "Laminar Flamelet Concepts in Turbulent Combustion," *Proceedings of the Combustion Institute*, Vol. 21, 1986, pp. 1231–120.

[43]Poinsot, T., and Veynante, D., *Theoretical and Numerical Combustion*, Edwards, Philadelphia, 2001.

[44]Turns, S., *An Introduction to Combustion*, McGraw–Hill, New York, 2000.

[45]Emmons, H. W., "Flow Discontinuities Associated with Combustion," *High Speed Aerodynamics and Jet Propulsion*, edited by H. W. Emmons, Vol. 3, Fundamentals of Gas Dynamics Princeton Univ. Press, Princeton, NJ, 1958, p. 584.

[46]Tsien, H. S., "Influence of Flame Front on the Flow Field," *Journal of Applied Mechanics*, Vol. 73, 1951, pp. 188–194.

[47]Fabri, J., Siestrunck, R., and Foure, L., "On the Aerodynamic Field of Stabilized Flames," *Fourth International Symposium on Combustion*, Combustion Inst., Pittsburgh, PA, 1953, pp. 443–450.

[48]Matalon, M., and Matkowsky, B., "Flames as Gas Dynamic Discontinuities," *Journal of Fluid Mechanics*, Vol. 124, 1982, pp. 239–260.

[49]Lieuwen, T., "Theoretical Investigation of Unsteady Flow Interactions With a Premixed Planar Flame," *Journal of Fluid Mechanics*, Vol. 435, May 2001, pp. 289–303.

[50]Clavin, P., and Williams, F., "Effects of Molecular Diffusion and of Thermal Expansion on the Structure and Dynamics of Premixed Flames in Turbulent Flows of Large Scale and Low Intensity," *Journal of Fluid Mechanics*, Vol. 116, 1982, pp. 251–282.

[51]Poinsot, T., Veynante, D., Bourienne, F., Candel, S., Esposito, E., and Surget, J., "Initiation and Suppression of Combustion Instabilities by Active Control," *Proceedings of the Combustion Institute*, Vol. 22, 1988, pp. 1363–1370.

[52]Sturgess, G., Hedman, P., Sloan, D., and Shouse, D., "Aspects of Flame Stability in a Research Dump Combustor," American Society of Mechanical Engineers, ASME Paper 94-GT-49, 1994.

[53]Chu, B. T., and Kovasnay, L. S. G., "Nonlinear Interactions in a Viscous, Heat Conducting, Compressible Gas," *Journal of Fluid Mechanics*, Vol. 3, 1958, pp. 494–514.

[54]Kovaszay, L. S. G., "Turbulence in Supersonic Flow," *Journal of the Aeronautical Sciences*, Vol. 20, No. 10, 1953, pp. 657–674.

[55]Jou, W.-H., and Menon, S., "Modes of Oscillation in a Nonreacting Ramjet Combustor Flow," *Journal of Propulsion and Power*, Vol. 6, No. 5, 1990, pp. 535–543.

[56]Clanet, C., Searby, G., and Clavin, P., "Primary Acoustic Instability of Flames Propagating in Tubes: Cases of Spray and Premixed Combustion," *Journal of Fluid Mechanics*, Vol. 385, April 1999, pp. 157–197.

[57]Ducruix, S., Durox, D., and Candel, S., "Theoretical and Experimental Determinations of the Transfer Function of a Laminar Premixed Flame," *Proceeding of the Combustion, Institute*, Vol. 28, 2000.

[58]Stone, C., and Menon, C., "Swirl Control of Combustion Instabilities in a Gas Turbine Combustor," *Proceedings of the Combustion Institute*, Vol. 29, 2002.

[59]Hegde, U. G., Reuter, D., Daniel, B. R., and Zinn, B. T., "Flame Driving of Longitudinal Instabilities in Dump Type Ramjet Combustors," *Combustion Science and Technology*, Vol. 55, No. 4–6, 1987, pp. 125–138.

[60]Smith, D., "An Experimental Study of Acoustically Excited, Vortex Driven Combustion Instability Within a Rearward Facing Step Combustor," Ph.D. Dissertation, California Inst. of Technology, Pasadena, CA, 1985.

[61]Straub, D. L., and Richards, G. A., "Effect of Fuel Nozzle Configuration on Premix Combustion Dynamics," American Society of Mechanical Engineers, ASME Paper 98-GT-492, 1998.

[62]Lieuwen, T., Torres, H., Johnson, C., and Zinn, B. T., "A Mechanism for Combustion Instabilities in Premixed Gas Turbine Combustors," *Journal of Engineering for Gas Turbines and Power*, Vol. 123, No. 1, 2001, pp. 182–190.

[63]Strahle, W., "On Combustion Generated Noise," *Journal of Fluid Mechanics*, Vol. 49, No. 2, 1971.

[64]Putnam, A. A, "Combustion Roar of Seven Industrial Burners," *J. Inst. Fuel*, Vol. 49, 1976, pp. 135–138.

[65]Pierce, A. D., *Acoustics: An Introduction to its Physical Principles and Applications*, Acoustical Society of America, New York, 1991.

[66]Lieuwen, T., and Zinn, B. T., "Application of Multipole Expansions to Sound Generation from Ducted Unsteady Combustion Processes," *Journal of Sound and Vibration*, Vol. 235, No. 3, 2000, pp. 405–414.

[67]Marble, F., and Candel, S., "Acoustic Disturbance from Gas Nonuniformity Convected Through a Nozzle," *Journal of Sound and Vibration*, Vol. 55, 1977, pp. 225–243.

[68]Clavin, P., "Dynamic Behavior of Premixed Flame Fronts in Laminar and Turbulent Flows," *Progress in Energy and Combustion Science*, Vol. 11, 1985.

[69]Mason, N., "Contribution to the Hydrodynamical Theory of Flame Vibration," *Proceedings of the Seventh International Congress on Applied Mechanics*, Vol. 2, 1948, pp. 187–199.

[70]Chu, B. T., "On the Generation of Pressure Waves at a Plane Flame Front," *Proceedings of the Combustion Institute*, Vol. 4, 1953, pp. 603–612.

[71]McIntosh, A. C., "Pressure Disturbances of Different Length Scales Interacting with Conventional Flames," *Combustion Science and Technology*, Vol. 75, No. 4–6, 1991, pp. 287–309.

[72]McIntosh, A. C., and Wilce, S. A., "High Frequency Pressure Wave Interaction with Premixed Flames," *Combustion Science and Technology*, Vol. 79, No. 1–3, 1991, pp. 141–155.

[73]Peters, N., and Ludford, G. S. S., "The Effect of Pressure Variations on Premixed Flames," *Combustion Science and Technology*, Vol. 34, 1983, pp. 331–344.

[74]Van Harten, A., Kapila, A., and Matkowsky, B. J., "Acoustic Coupling of Flames," *SIAM Journal on Applied Mathematics*, Vol. 44, No. 5, 1984, pp. 982–995.

[75]Keller, D., and Peters, N., "Transient Pressure Effects in the Evolution Equation for Premixed Flame Fronts," *Theoretical Computational Fluid Dynamics*, Vol. 6, 1994, pp. 141–159.

[76]Ledder, G., and Kapila, A. K., "The Response of Premixed Flames to Pressure Perturbations," *Combustion Science and Technology*, Vol. 76, No. 1–3, 1991, pp. 21–44.

[77]McIntosh, A. C., "Deflagration Fronts and Compressibility," *Philosophical Transactions of the Royal Society of London, Series A: Mathematical and Physical Sciences*, Vol. 357, 1999, pp. 3523–3538.

[78]Joulin, G., "On the Response of Premixed Flames to Time Dependent Stretch and Curvature," *Combustion Science and Technology*, Vol. 97, No. 1–3, 1994, pp. 219–229.

[79]Huang, Z., Bechtold, J., and Matalon, M., "Weakly Stretched Premixed Flames in Oscillating Flows," *Combustion Theory Modeling*, Vol. 2, 1998, pp. 115–133.

[80]Im, H. G., and Chen, J. H., "Effects of Flow Transients on the Burning Velocity of Laminar Hydrogen/Air Premixed Flames," *Proceedings of the Combustion Institute*, Vol. 28, 2000, pp. 1833–1840.

[81]Malard, E., and Le Chatelier, H., "Recherches experimentelles et theoriques sur la combustion des mélanges gazeux explosives," *Annls Mines Paris*, Partie Scientifique et Technique, Ser. 8, No. 4, 1883, p. 274.

[82]Kaskan, W., "An Investigation of Vibrating Flames," *Proceedings of the Combustion Institute*, Vol. 4, 1953, pp. 575–591.

[83]Searby, G., "Acoustic Instability in Premixed Flames," *Combustion Science and Technology*, Vol. 81, No. 4–6, 1992, pp. 221–231.

[84]Clavin, P., Pelce, P., and He, L., "One-Dimensional Vibratory Instability of Planar Flames Propagating in Tubes," *Journal of Fluid Mechanics*, Vol. 216, 1990, pp. 299–322.

[85]Markstein, G., "Flames as Amplifiers of Fluid Mechanical Disturbances," *Proceedings of the Sixth National Congress on Applied Mechanics*, 1970, pp. 11–33.

[86]Searby, G., and Rochwerger, D., "A Parametric Acoustic Instability in Premixed Flames," *Journal of Fluid Mechanics*, Vol. 231, 1991, pp. 529–543.

[87]Pelce, P., and Rochwerger, D., "Vibratory Instability of Cellular Flames Propagating in Tubes," *Journal of Fluid Mechanics*, Vol. 239, 1992, pp. 293–307.

[88]Vaezi, V., and Aldredge, R., "Laminar Flame Instabilities in a Taylor-Couette Combustor," *Combustion and Flame*, Vol. 121, 2000, pp. 356–366.

[89]Bychkov, V., "Analytical Scalings for Flame Interaction with Sound Waves," *Physics of Fluids*, Vol. 11, No. 10, 1999, pp. 3168–3173.

[90]Vaezi, V., and Aldredge, R. C, "Influences of Acoustic Instabilities on Turbulent-Flame Propagation," *Experiments in Thermal and Fluid Science*, Vol. 20, 2000, pp. 162–169.

[91]Lieuwen, T., "Theory of High Frequency Acoustic Wave Scattering by Turbulent Flames," *Combustion and Flame*, Vol. 126, No. 1-2, 2001, pp. 1489–1505.

[92]Lieuwen, T., "Analysis of Acoustic Wave Interactions with Turbulent Premixed Flames," *Proceedings of the Combustion Institute*, Vol. 29, 2002.

[93]Lieuwen, T., Neumeier, Y., and Rajaram, R., "Measurements of Incoherent Acoustic Wave Scattering from Turbulent Premixed Flames," *Proceedings of the Combustion Institute*, Vol. 29, 2002.

[94]Park, Y., and Vlachos, D., "Isothermal Chain-Branching, Reaction Exothermicity, and Transport Interactions in the Stability of Methane/Air Mixtures," *Combustion and Flame*, Vol. 114, No. 1–2, 1998, pp. 214–230.

[95]Kalamatianos, S., Park, Y., and Vlachos, D., "Two-Parameter Continuation Algorithms for Sensitivity Analysis, Parametric Dependence, Reduced Mechanisms, and Stability Criteria of Ignition and Extinction," *Combustion and Flame*, Vol. 112, No. 1–2, 1998, pp. 45–61.

[96]Richards, G. A., Morris, G. J., Shaw, D. W., Keely, S. A., and Welter, M. J., "Thermal Pulse Combustion," *Combustion Science and Technology*, Vol. 94, 1993, pp. 75–85.

[97]Janus, M. C., and Richards, G., "Results of a Model for Premixed Combustion Oscillations," *Proceedings of the 1996 Meeting of the American Flame Research Council*, 1996.

[98]Lieuwen, T., Neumeier, Y., and Zinn, B. T., "The Role of Unmixedness and Chemical Kinetics in Driving Combustion Instabilities in Lean Premixed Combustors," *Combustion Science and Technology*, Vol. 135, 1998, pp. 193–211.

[99]Park, S., Annaswamy, A., and Ghoniem, A., "Heat Release Dynamics Modeling of Kinetically Controlled Burning," *Combustion and Flame*, Vol. 128, No. 5, 2002, pp. 217–231.

[100]Park, S., Annaswamy, A., Ghoniem, A., Dynamic Characteristics of Kinetically Controlled Combustion and their Impact on Thermoacoustic Instability, *Proceedings of the International of Collegius in the Dynamics of Explosions and Reactive Systems*, 2001.

[101]Becker, R., and Gunther, R., "The Transfer Function of Premixed Turbulent Jet Flames," *Proceedings of the Combustion Institute*, Vol. 13, 1970, pp. 517–526.

[102]Merk, H. J., "An Analysis of Unstable Combustion of Premixed Gases," *Proceedings of the Combustion Institute*, Vol. 6, 1956, pp. 500–512.

[103]Matsui, Y., "An Experimental Study on Pyro-Acoustic Amplification of Premixed Laminar Flames," *Combustion and Flame*, Vol. 43, 1981, pp. 199–209.

[104]Marble, F. E., and Candel, S. M., "An Analytical Study of the Non-Steady Behavior of Large Combustors," *Proceedings of the Combustion Institute*, Vol. 17, 1978, pp. 761–769.

[105]Subbaiah, M. V., "Nonsteady Flame Spreading in Two Dimensional Ducts," *AIAA Journal*, Vol. 21, No. 11, 1983, pp. 1557–1564.

[106]Poinsot, Thierry, Candel, S. M., and Sebastien, M., A Nonlinear Model for Ducted Flame Combustion Instabilities, *Combustion Science and Technology*, Vol. 61, No. 4–6, 1988, pp. 121–151.

[107]Yang, V., and Culick, F. E. C., "Analysis of Low Frequency Combustion Instabilities in a Laboratory Ramjet Combustor," *Combustion Science and Technology*, Vol. 45, pp. 1–25.

[108]Riley, J., Golschmidt, V. W., Leonard, R. G., and Baade, P. K., "Noise Generation of Gas Flames due to Feedback Excited Oscillations," *Proceedings of the 3rd Conference on Natural Gas Research*, 1974.

[109]Boyer, L., and Quinard, J., "On the Dynamics of Anchored Flames," *Combustion and Flame*, Vol. 82, 1990, pp. 51–65.

[110]Baillot, F., Durox, D., and Prud'homme, R., "Experimental and Theoretical Study of a Premixed Vibrating Flame," *Combustion and Flame*, Vol. 88, No. 2, 1992, pp. 149–168.

[111]Fleifel, M., Annaswamy, A. M., Ghoniem, Z. A., and Ghoniem, A. F., "Response of a Laminar Premixed Flame to Flow Oscillations: A Kinematic Model and Thermoacoustic Instability Results," *Combustion and Flame*, Vol. 106, No. 4, 1996, pp. 487–510.

[112]Ferguson, D., Richards, G., Woodruff, S., Bernal, S., and Gautam, M., "Effect of Surface Area Variation on Heat Release Rates in Premixed Flames, *Proceedings of the Second Joint Meeting of the US Sections of the Combustion Institute*, 2001.

[113]Lee, D. H., and Lieuwen, T., "Acoustic Nearfield Characteristics of a Conical, Premixed Flame," *Journal of the Acoustical Society of America*, Vol. 113, No. 1, 2003, pp. 167–177.

[114]Keller, J., Vaneveld, L., Korschelt, D., Hubbard, G., Ghoniem, A., Daily J., and Oppenheim, A., "Mechanism of Instabilities in Turbulent Combustion Leading to Flashback," *AIAA Journal*, Vol. 20, 1982, p. 254.

[115]Lieuwen, T., and Neumeier, Y., "Nonlinear Pressure-Heat Release Transfer Function Measurements in a Premixed Combustor," *Proceedings of the Combustion Institute*, Vol. 29, 2002.

[116]Rhode, M. A., Rollins, R. W., Markworth, A. J., Edwards, K. D., Nguyen, K., Daw, C. S., and Thomas, J. F., "Controlling Chaos in a Model of Thermal Pulse Combustion," *Applied Physics*, Vol. 78, No. 4, 1995, p. 2224.

[117]Ashurst, W., Sivashinsky, G., and Yakhot, V., "Flame Front Propagation in Nonsteady Hydrodynamic Fields," *Combustion Science and Technology*, Vol. 62, 1988, pp. 273–284.

[118]Ashurst, W., and Sivashinsky, G., "On Flame Propagation through Periodic Flow Fields," *Combustion Science and Technology*, Vol. 80, 1991, pp. 159–164.

[119]Joulin, G., and Sivashinsky, G., "Pockets in Premixed Flames and Combustion Rate," *Combustion Science and Technology*, Vol. 77, 1991, pp. 329–335.

[120]Aldredge, R., "The Propagation of Wrinkled, Premixed Flames in Spatially Periodic Shear Flow," *Combustion and Flame*, Vol. 90, No. 2, 1992, pp. 121–133.

[121]Bourehla, A., and Baillot, F., "Appearance and Stability of a Laminar Conical Premixed Flame Subjected to an Acoustic Perturbation," *Combustion and Flame*, Vol. 114, No. 3, 1998, pp. 303–318.

[122]Baillot, F., Bourehla, A., and Durox, D., "The Characteristic Method and Cusped Flame Fronts," *Combustion Science and Technology*, Vol. 112, 1996, pp. 327–350.

[123]Durox, D., Baillot, F., Searby, G., and Boyer, L., "On the Shape of Flames Under Strong Acoustic Forcing: A Mean Flow Controlled by an Oscillating Flow," *Journal of Fluid Mechanics*, Vol. 350, 1997, pp. 295–310.

[124]Blackshear, P., "Driving Standing Waves by Heat Addition," *Proceedings of the Combustion Symposium*, Vol. 4, 1953, pp. 553–566.

[125]Saitoh, T., and Otsuka, Y., "Unsteady Behavior of Diffusion Flames and Premixed Flames for Counter Flow Geometry," *Combustion Science and Technology*, Vol. 12, No. 4–6, 1976, pp. 135–146.

[126]Dowling, A. P., "Nonlinear Self-Excited Oscillations of a Ducted Flame," *Journal of Fluid Mechanics*, Vol. 346, 1997, pp. 271–290.

[127]Dowling, A. P., "A Kinematic Model of a Ducted Flame," *Journal of Fluid Mechanics*, Vol. 394, 1999, pp. 51–72.

[128]Peracchio, A. A., and Proscia, W. M., "Nonlinear Heat Release/Acoustic Model for Thermo-Acoustic Instability in Lean Premixed Combustors," American Society of Mechanical Engineers, ASME Paper 98-GT-269, 1998.

[129]Cho, J. H., and Lieuwen, T., "Modeling the Response of Premixed Flames to Mixture Ratio Perturbations," American Society of Mechanical Engineers, ASME Paper GT2003-38089, 2003.

[130]Abu-Off, G. M., and Cant, R. S., "Reaction Rate Modeling for Premixed Turbulent Methane-air Flames," Proc. Joint Meeting of Spanish, Portuguese, Swedish and British Sections of the Combustion Institute, 1996.

[131]Hubbard, S., and Dowling, A. P., "Acoustic Instabilities in Premix Burners," AIAA Paper 98-2272, 1998.

[132]Polifke, W., Poncet, A., Paschereit, C. O., and Dobbeling, K., "Reconstruction of Acoustic Transfer Matrices by Instationary Computational Fluid Dynamics," *Journal of Sound and Vibration*, Vol. 245, No. 3, 2001, pp. 483–510.

[133]Wee, D., Park, S., Annaswamy, A., and Ghoniem, A., "Reduced Order Modeling of Reacting Shear Flow," AIAA Paper 2002—0478, 2002.